THE MINIMAX OPTIMAL CONTROL PROBLEM FOR DINAMIC SYSTEM WITH PARAMETER AND under conditions of INDETERMINACY

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ABSTRACT
In the paper we consider one class dynamic control system with discrete parameter and under conditions of indeterminacy initial data. The minimax control problem for this system is researched. For this non-smooth control problem the necessary and sufficient conditions for optimality are obtained.

Keywords: control system, discrete parameter, non-smooth functional, minimax, control problem, conditions of optimality.

INTRODUCTION
The issues of decision-making in the economic planning and organization of production, in the design of technical devices and process control lead to various optimization problems [1,2]. Non-smooth optimization problems constitute a special class of mathematical models of such problems. They are often represented with non-smooth objective functions.

One of the approaches used in decision-making in conditions of incomplete information about the initial data of the system and external influences is the principle of optimization by the minimax criterion, which assumes obtaining a guaranteed value of the management quality criterion [3]. This usually leads to problems of optimizing a non-smooth function of the maximum or minimum type [4,5].

In the research of system models under conditions of information constraints, the methods of estimating the reachability set and predicting the phase state of the system, the conditions of guaranteed control, etc. are of great interest. They are closely related to the problems of controlling an ensemble of trajectories [6-9] and are solved by methods of non-smooth optimization problems for dynamic control systems.

As a result of studies of non-smooth optimization models, special methods for solving them have been developed, and sections of non-smooth and multi-valued analysis are being developed [1,3,4,6].

In this paper, we consider a dynamic control system with a discrete parameter under conditions of incomplete information about the initial state. The goal of management is to achieve a guaranteed result under such conditions of inaccuracy of information. A terminal functional of the minimum function type is considered as a criterion for evaluating the quality of management. Necessary and sufficient optimality conditions are obtained. They develop research of work [10]

STATEMENT OF THE PROBLEM
Consider a dynamic control system with a parameter of the form

$$\dot{x} = A(t,y)x + b(t,u,y), t \in T = [t_0, t_1],\quad u \in V,\ y \in Y, \qquad (1)$$

where x – is state n -vector, u – is control m vector, y – k - dimensional parameter, $A(t,y)$ – $n \times n$ -matrix, $b(t,u,y) \in R^n$. The initial state of the system is inaccurate, i.e. $x(t_0) \in D$, where D – convex compact subset R^n ; V – compact space R^m ; the parameter y takes discrete values, i.e.

$Y=\{y_1,y_2,...,y_q\}$. With respect to the right side of equation (1), we assume that the following conditions are met:

1) the elements of the matrix $A(t,y)$ are summable by $t\in T$ for each $y\in Y$;

2) the mapping $(t,u,y)\to b(t,u,y)$ is measurable over $t\in T$ and continuous over $u\in V$, for every $y\in Y$, with $\|b(t,u,y)\|\le \beta(t)$, $\beta(\cdot)\in L_1(T)$.

The admissible controls for system (1) are each measurable bounded m-vector function $u=u(t)$, $t\in T$, which take values from V almost everywhere on T.

Let: U_T – the set of all admissible controls; $H_T(u,y)$ –the set of all absolutely continuous solutions $x=x(t,u,x_0,y)$ of equation (1) with the initial condition $x(t_0)=x_0\in D$ for a given admissible control $u\in U_T$ and parameter $y\in Y$; Under given conditions, $H_T(u,y)$ is a compact set in the space of continuous n-vector functions $C^n(T)$.

Let the quality of control of a dynamical system be evaluated by a non-smooth terminal functional $J(u,x_0)=\inf_{l\in L}\sum_{y\in Y}(P(y)x(t_1,u,x_0,y),l)$, where $P(y)$ is an $s\times n$ matrix, and L is a bounded set of R^s. Given the inaccuracies of setting the initial state of the system (1), the goal of management can be considered to achieve a guaranteed value of the quality criterion $J(u,x_0)$. In other words, for system (1), consider the following minimax problem:

$$J(u)\equiv \max_{x(\cdot)\in H_T(u,y),y\in Y}\ \inf_{l\in L}\sum_{y\in Y}(P(y)x(t_1),l)\to \min, u\in U_T. \qquad (2)$$

We will study the necessary and sufficient optimality conditions for the minimax problem (2).

METHODS AND RESULTS OF THE STUDY

Consider a set consisting of the ends of all the trajectories of $x(\cdot)\in H_T(u,y)$ at time $t_1>t_0$:

$$X_T(t_1,u,y)=\{\xi\in R^n \mid \xi=x(t_1), x(\cdot)\in H_T(u,y)\}.$$

Due to the results of [6], $X_T(t_1,u,y)$ is a convex compact set of R^n. Using the minimax theorem known from convex analysis, we obtain that the equality is valid

$$\max_{x(\cdot)\in H_T(u,y),y\in Y}\ \inf_{l\in L}\sum_{y\in Y}(P(y)x(t_1),l)=\inf_{l\in coL}\sum_{y\in Y}\ \max_{x(\cdot)\in H_T(u,y),y\in Y}(P(y)x(t_1),l)\ \forall u\in U_T.$$

Therefore, the minimax problem (2) can be written as follows:

$$\inf_{l\in coL}\sum_{y\in Y}C(P(y)X(t_1,u,y),l)\to \min, u\in U_T, \qquad (3)$$

where $C(PX,l)=\sup_{\xi\in X}(P\xi,l)$ – support function of the set PX, coL – convex hull of a set L. In that way, the minimax problem (2) is reduced to the problem of repeated minimize (3). View this problem clear that it is a task management terminal state of the ensemble of trajectories of a dynamical system (1) with imprecisely specified initial condition.

Consider the function $\psi(t,y,l)=F_y'(t_1,t)P'(y)l$. The reference function $C(P(y)X(t_1,u,y),l))$ of the set $P(y)X(t_1,u,y)$ has the representation:

$$C(P(y)X_T(t_1,u,y),l)=C(D,\psi(t_0,y,l)+\int_{t_0}^{t_1}(b(t,u(t),y),\psi(t,y,l)dt.$$

where $F_y(t,\tau)$ – the fundamental matrix of solutions to the equation $\dot{x}=A(t,y)x$, i.e. $\frac{\partial F_y(t,\tau)}{\partial t}=A(t,y)F_y(t,\tau)$, $t\in T,\tau\in T$, $F_y(\tau,\tau)=E$, $E-$ singular $n\times n$ - matrix.

Let's introduce the function

$$\gamma(l)=\sum_{y\in Y}C(D,\psi(t_0,y),l)+\int_{t_0}^{t_1}\min_{v\in V}\sum_{y\in Y}C(b(t,v,y),\psi(t,y,l))dt, l\in coL.$$

Theorem. For optimality control $u^0(\cdot)$ in problem (2), the existence of a global minimum point $l^0\in coL$ of the function $\gamma(l), l\in coL$ is necessary and sufficient, and the condition is satisfied

$$\min_{v\in V}\sum_{y\in Y}(b(t,v,y),\psi(t,y,l^0)))=\sum_{y\in Y}(b(t,u^0(t),y),\psi(t,y,l^0)) \text{ i.e. on } T. \qquad (4)$$

CONCLUSION

In this paper, we study the problem of controlling an ensemble of trajectories of a system formulated in the form of a non-smooth control problem of the minimax type. Necessary and sufficient optimality conditions are obtained for this problem. They provide a theoretical justification for the method of constructing a solution to problem (2) by solving finite-dimensional problems of the form (4).

REFERENCES

1. Clarke F. Optimization and non-smooth analysis, Willey & Sons, Ney York, 1983.
2. Chernomorov G. A. Theory of decision-making. - Novocherkassk: 2002.
3. Kane V. N. Optimization of control systems by minimax criterion. - Nauka Moscow, 1985.
4. Demyanov V. F., Rubinov A.M. Fundamentals of non-smooth analysis and quasi-differential calculus, Nauka, Moscow, 1990.
5. Otakulov S., Khaydarov T. T. Properties of maximum and minimum type functions and their application to non-smooth optimization problems. Tadqiqot.uz. Physical and mathematical sciences. Vol. 3, Issue 1. pp.32-37. Doi: http://dx.doi.org/10.26739/2181-0656-2020-3-7

6. Otakulov S. Problems of controlling an ensemble of trajectories of differential inclusions. Monograph. Lambert Academic Publishing, 2019.
7. Polovinkin E. S. Set-valued analysis and differential inclusions, Fizmatlit, Moskow, 2015.
8. Otakulov S., Kholiyarova F.Kh. About conditions of controllability of ensemble trajectories of differential inclusion with delay. International Journal of Statistics and Applied Mathematics.vol.5(3), 2020.-pp.59–65.
9. Otakulov S., Rahimov B. Sh. About the property of controllability an ensemble of trajectories of differential inclusion. International Engineering Journal for Research & Development(IEJRD). Vol.5, issue 4, 2020. pp.1-9.
10. Otakulov S., Sobirova G.D. About time control problem for ensemble of trajectories of differential inclusion. Science and Education. Vol.1, Issue 9, 2020. pp. 9-17.
11. Otakulov S.,Haydarov T.T. The non-smooth control problem for dynamic system with parameter under conditions of incomplete initial date. International Conference On Innovation Perspectives, Psychology and Social Studiees (ICIPPCS-2020). International Enjineering Journal for Research & Development. pp.211-214. DOI: https://doi.org/10.17605/OSF.10/BN39W