



UNSTATIONARY FLOW OF A VISCOELASTIC LIQUID IN A PLANE CHANNEL

¹Navruzo Kuralbay, ²Khujatov Nurbek, ³Begjanov Amirbek

Professor Urgench state university¹, PhD student, Urgench state university², PhD student, Urgench state university³
qurol_46@mail.ru¹, khujatov@bk.ru²

ABSTRACT

This article is devoted to the study of the stationary flow of a viscous fluid in a flat channel with permeable walls. As is known, in a pulsating flow in pipelines, a pressure wave propagates through its walls, which is called a pulse wave. This wave, as it moves away from the initial cross section, gradually weakens and practically attenuates at the ends. In the present work, stationary flows of a viscous fluid in a plane channel with permeable walls are considered to be sufficiently long for the channel. The problems are solved and calculation formulas for determining the longitudinal and transverse speeds are obtained. Formulas are also found and numerical calculations are made that determine the change in pressure and fluid flow in the direction of the main stream for various values of the permeability coefficient. It is shown that at lower values of the coefficient of permeability, the pressure is distributed along the flow according to linear laws, and the fluid flow rate remains almost constant along the channel length. Other values of the permeability coefficient, it is characteristic that with an increase in the permeability coefficient of the pressure distribution differs significantly from the linear distribution, with the maximum deviation in the middle of the channel. The fluid flow rate with an increase in the permeability coefficient increases several times in the initial section compared to the fluid flow rate, in the same pressure drop in the flow of a viscous fluid in a flat channel with impermeable walls.

Keywords: viscosity , stationarity , flat, permeability, hydraulic resistance, longitudinal speed , lateral speed, pressure, density , pressure.

Studies [3-5] show that solving the problem of unsteady flow of a viscoelastic fluid in pipelines leads to significant mathematical difficulties. Therefore, when solving problems, methods of simplification are used, or the problem [7,8] is solved in a one-dimensional formulation with lumped velocities over the pipe section. In the majority of works [3-5], it is argued that transient processes occur during unsteady flow, which significantly depend on the properties of the fluids flowing in them. In this article, specific problems are solved about the unsteady flow of an elastic-viscous fluid in a flat channel and transient processes are analyzed. In addition, when solving one-dimensional problems, the data obtained are not only of great importance for the detection of new hydrodynamic effects, but also are a reliable source for comparison with the results of studying an elastic-viscous fluid in a more complex formulation. On the basis of the rheological models of an elastic-viscous fluid proposed in [4,5], we will solve non-stationary problems in pipes and channels, where the fluid is assumed to be elastic-viscous and incompressible, and its motion is laminar and axisymmetric.

In this case, the movements of the liquid in the pipes, taking into account its rheological properties, are described by simplified equations of the following form

$$\begin{cases} \rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}(\tau_{xy}), & \frac{\partial p}{\partial y} = 0, \\ \tau_{xy} = \sum_{k=1}^{\infty} \tau_{k,xy}, & \frac{\partial \tau_{k,xy}}{\partial t} + \frac{g_k}{\lambda_k} \tau_{k,xy} = p_k \frac{\partial u}{\partial t}, \\ \frac{\partial p_k}{\partial t} + \frac{g_k}{\lambda_k} p_k = \frac{\eta_k}{\lambda_k^2} f_k. \end{cases} \quad (1)$$

To solve the system of equations (1), it is necessary to formulate the initial and boundary conditions. We assume that at, the liquid in the initial state is assumed to be "at rest", i.e.

$$u = 0, \quad \frac{\partial p}{\partial x} = 0 \quad \text{at } t = 0 \quad (2)$$

$$u = 0, \quad \text{at } y = h, \quad \frac{\partial u}{\partial y} = 0 \quad \text{at } y = 0 \quad u \neq 0, \quad \frac{\partial p}{\partial x} \neq 0 \quad \text{at } t > 0 \quad (3)$$

Boundary value problems for equations (1) with boundary and initial conditions (2) and (3) can be solved in the final form only for flows, when $f_k = 1$, $g_k = 1$ in equation (1), and for a fluid with a relaxation time spectrum

$$\lambda_k = \frac{\lambda}{k^\alpha}, \quad \eta_k = \frac{\eta}{\xi(\alpha)k^\alpha}.$$

The most complex fluid flows with a nonlinear spectrum of relaxation times require the use of a cumbersome apparatus of mathematical physics or a numerical method. Linearized equations (1) and initial and boundary conditions (2), (3) using the Laplace-Carson transformation [6], in time, taking into account the initial conditions, can be written as:

$$\begin{cases} \frac{\partial \bar{\tau}_{xy}}{\partial y} - \rho s \bar{u} = \frac{\partial \bar{p}}{\partial x}, \\ \frac{\partial \bar{p}}{\partial y} = 0, \quad \bar{\tau}_{xy} = \sum_{k=1}^N \bar{\tau}_{k,xy}; \\ s \bar{\tau}_{k,xy} + \frac{1}{\lambda_k} \bar{\tau}_{k,xy} = \frac{\eta_k}{\lambda_k} \frac{\partial \bar{u}}{\partial y} \end{cases} \quad (4)$$

From equation (4) one can find $\bar{\tau}_{rx}$:

$$\bar{\tau}_{xy} = \sum_{k=1}^N \frac{\eta_k}{1 + s\lambda_k} \frac{\partial \bar{u}}{\partial y} = \eta(s) \frac{\partial \bar{u}}{\partial y}, \quad (5)$$

Where

$$\sum_{k=1}^N \frac{\eta_k}{1 + s\lambda_k} = \eta(s)$$

Substituting the obtained expression (5) into equation (4), we obtain

$$\frac{\partial^2 \bar{u}}{\partial y^2} - \frac{\rho_0 s}{\eta(s)} \bar{u} = \frac{1}{\eta(s)} \frac{\partial \bar{p}}{\partial x}. \quad (6)$$

In equation (6) $\frac{\partial \bar{p}}{\partial x}$ does not depend on the channel width, so its solution can be obtained in the form of a

trigonometric function. It is believed that the variable x is a "frozen" parameter in the equation:

$$\bar{u} = \frac{1}{\rho s} \left(-\frac{\partial \bar{p}}{\partial x} \right) \left(1 - \frac{\cos \left(i \sqrt{\frac{\rho_0 s}{\eta(s)}} y \right)}{\cos \left(i \sqrt{\frac{\rho_0 s}{\eta(s)}} h \right)} \right). \quad (7)$$

This expression only has simple poles

$$s = 0, \quad s = -v \frac{\bar{s}_{ni}}{h^2}$$

where \bar{s}_{ni} is the root of the transcendental equation

$$\bar{s} + \bar{\eta}(\bar{s}) \left(\frac{2p+1}{2} \right)^2 \pi^2 = 0: \quad (8)$$

Here

$$\bar{\eta}(\bar{s}) = \frac{1}{\xi(\alpha)} \sum_{k=1}^{\infty} \frac{1}{k^\alpha - EL\bar{s}}. \quad (9)$$

is the Shulman-Husid rheological equation, where $EL = \frac{v\lambda}{h^2}$. It can be solved analytically in two cases,

when $|\lambda s| \ll 1$ or $|\lambda s| \gg 1$.

In the first case, when $|\lambda s| \ll 1$ the rheological equation turns into the equation of a Newtonian fluid, i.e.

$$\bar{\eta}(\bar{s}) = \frac{1}{\xi(\alpha)} \sum_{k=1}^{\infty} \frac{1}{k^\alpha - EL\bar{s}} \approx \frac{1}{\xi(\alpha)} \sum_{k=1}^{\infty} \frac{1}{k^\alpha} = 1. \text{ Hence, } \eta(s) = \eta \bar{\eta}(\bar{s}) = \eta \text{ in this case, all}$$

the results obtained for the Newtonian fluid are true for an elastic-viscous fluid when the rheological equations are given in the form of the Shulman-Husid model.

In the second case, when $|\lambda s| \gg 1$ (8) is the transcendental equation when the expression $\bar{\eta}(\bar{s})$ is

$$\text{replaced by its asymptotic expression } \bar{\eta}(\bar{s}) = \frac{\pi}{\xi(\alpha)\alpha \sin(\pi/\alpha)(\lambda s)^{(1-1/\alpha)}}$$

Then (8) at $\alpha = 2$ has the form

$$\bar{s} + \frac{\pi}{2\xi(2)(-EL\bar{s})^{\frac{1}{2}}} \frac{(2p+1)^2}{2} \pi^2 = 0:$$

$$\text{or } \bar{s}_{in} = \frac{\pi^2}{4\sqrt[3]{\xi^2(2)EL}} (2n+1)^{4/3} \left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$$

The final solution would be like this

$$\frac{u(0,t)}{u_{0max}} = \left[1 - 32 \sum_{n=0}^{\infty} (-1)^n \frac{\sqrt[3]{\xi^2(2)EL}}{3(2n+1)^2 \pi^3 \sqrt[3]{2n+1}} e^{-\frac{v}{h^2 \bar{s}_n t}} \right]$$

Using the obtained solution, numerical calculations can be made for the Shulman-Husid model.

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