# MATHEMATICS TEACHING AND LEARNING

**Rakhimova Dildora Mukhammedovna**
25 school teachers of Bagat district of Khorezm region, Uzbekistan


## ABSTRACT

This paper focuses on advances in the study of mathematics teaching and learning since the publication of the first edition of the *Handbook of Educational Psychology* (Berliner & Calfee, editors) in 1996. Because of the scope of the review, comprehensive coverage is not possible. In what follows I have chosen to focus thematically on major areas in which progress has been made or where issues at the boundaries of theory and practice are controversial.[1] These areas include: research focusing on issues of teacher knowledge and aspects of professional development; issues of curriculum development, implementation, and assessment; issues of equity and diversity; and issues of learning in context(s). The chapter concludes with a discussion of the state of the field and its contextual surround. To provide the context for what follows, this chapter begins with a brief historical review. The story line starts at the turn of the 20th century, with increasing attention given to more current trends. The topics addressed include demographics, curriculum content, and the philosophical and epistemological underpinnings of curricula, research methods, and findings.


***Key Words. Method, formula, organization.***

PUFM is fundamentally mathematical – the core ideas are about mathematical structure. But it is also fundamentally pedagogical, with an organization aimed at meaning-making and deep understanding. In this sense, the PUFM possessed by an accomplished teacher overlaps with, but is different from, that of an accomplished mathematician. There are likely to be aspects of elementary mathematics such as rational number (fractions) that any mathematician knows, and that a highly accomplished teacher does not know – for example, the formal definition of the rational numbers as equivalence classes of ordered pairs of integers. But, there are also aspects of elementary mathematics that teachers with PUFM possess, and professional mathematicians do not. These include having a substantial number of ways of giving meaning to mathematical operations and concepts, and seeing and fostering connections among them. PUFM represents a deeper, more connected understanding of elementary mathematical sense-making than mathematicians are likely to know. It is a different (though related) form of knowledge.

### *Magdalene Lampert's "Teaching Problems and the Problem of Teaching"*

In *Teaching Problems and the Problem of Teaching*, Lampert (2001) takes on the extraordinarily difficult challenge of unraveling the complexities of teaching – of portraying the complex knowledge, planning, and decision-making in which she engaged, over the course of a year, as she taught a class in fifth grade mathematics. This book is an eloquent and elegant antidote to simplistic views of the teaching process. Lampert writes:One reason teaching is a complex practice is that many of the problems a teacher must address to get students to learn occur simultaneously, not one after another. Because of this simultaneity, several different problems must be addressed by a single action. And a teacher's actions are not taken independently; there are

[1] This approach, like any approach to mapping out a huge territory, results in some unfortunate omissions. Many fine pieces of work, specifically, many studies that focus on learning and conceptual growth in particular mathematical topic areas, are not discussed here. Nor is the role of technology in mathematics learning. Readers with specific interests in these topics will want to consult the forthcoming *Second Handbook of Research on Mathematics Teaching and Learning* (Lester, in preparation).

inter-actions with students, individually and as a group.… When I am teaching fifth-grade mathematics, for example, I teach a mathematical idea or procedure to a student while also teaching that student to be civil to classmates and to me, to complete the tasks assigned, and to think of herself or himself and everyone else in the class as capable of learning, no matter what their gender, race, or parents' income. As I work to get students to learn something like "improper fractions," I know I will also need to be teaching them the meaning of division, how division relates to other operations, and the nature of our number system. (Lampert, 2001, p. 2).

Lampert views and portrays her teaching through multiple lenses. She begins close up, with a view of a specific lesson on rate. One day her students enter the classroom after recess and find the following problem on the board:

Condition: A car is going 55 mph. Make a diagram to show where it will be:

A. after an hour
B. after two hours
C. after half an hour
D. after 15 minutes.

Lampert describes individual students in the class, and what they began to do with the problem. She shows the varied representations students made, which provide insights into the students' current state of understanding. She then describes her work in orchestrating a communal discussion of the problem, and the representation she used. She then zooms in on one particular interaction, which occurred when a student wrote an answer to part D of the problem on the board that she could not understand. As she often does, she asked the class if others could explain where that answer might have come from. She calls on a student whom she thinks will do so, but that student asks instead if she can explain her own solution. This raises a dilemma for Lampert. Which train of reasoning does she follow? In doing so, (a) whom does she run the risk of enfranchising or disenfranchising, and what implications will this have for the power relationships developing in the classroom? (b) which aspects of the mathematics will be publicly aired, helping other students to connect not only to the "correct" answer but to think through the various ways of understanding the problem? As she wrestles with this, the first student asks to change what he has written. He does, and the number he places on the board is close to the right answer. Now Lampert faces yet another choice. How can she "unpack" this student's thinking, so the class can see how and why he arrived at it, and orchestrate a classroom conversation that will result in the student and the class figuring out the right answer? How can she do so in a way that teaches meta-lessons about reviewing and verifying one's work, that connects to as many of the students' understandings as possible, and that reinforces the classroom's norms of respectful and substantive mathematical interactions?

All this and more happens in one segment of one lesson. And, a lesson is a very small part of a year (which, it should be noted, is 10% of a fifth-grader's life-to-date, so personal as well as intellectual development is a very big issue!). The art of Lampert's book is that she presents the incidents in enough detail to allow one to experience them, at least vicariously; then she steps back, providing an analytic commentary on what took place. Over the course of the book, Lampert displays and reflects upon multiple aspects of her teaching, at various levels of grain size. In an early chapter, she presents her reflections and notes on how to get the year started. She identifies her major goals. She compiles a list of productive activities. She views the year through a content lens – students will need to learn the concept of fraction, long division and multiplication, and more. She considers issues related to "learning the practice of mathematics, things like: revision; hypothesizing; giving evidence, explanation; representation." There are issues of physical environment. These are planned in some detail, and then revised in response to ongoing reality – who the students are, and how things seem to be working. Here too, Lampert presents a substantial amount of detail. If you want students to learn how to make conjectures public, and then to work through those conjectures respectfully (including challenging others' ideas and/or retracting one's own when it turns out not to be right), one must to pick problems that will support rich interactions, and work on establishing the right classroom norms.

Some of Lampert's discussions, like that of the automobile rate problem, describe teaching in-the-moment. Some involve planning to establish both mathematical content and classroom culture. Some involve making design choices. Given a particular topic, which examples will be accessible to students, will support rich reasoning, will bring the students to confront the central conceptual issues in the domain? How does one develop appropriate pedagogies for independent work (one goal is to have students develop as independent thinkers), for small group work (one goal is to have students learn to interact well and closely with each other, and profit from those interactions), and whole-class discussions? How can the curriculum be arranged to help students see mathematical connections? How can she simultaneously "cover" the material mandated by her school district and state curriculum frameworks? These are serious design and implementation issues.

As noted above, classroom considerations for a fifth grade teacher go far beyond issues of content. A chapter of Lampert's book is devoted to "teaching students to be people who study in school." How does one realize goals such as "teaching intellectual courage, intellectual honesty, and wise restraint" – having students learn to be willing to take considered risks, be ready to change their position with regard to an issue on the basis of new evidence, but weighing evidence carefully before taking or revising a position? How does one define accomplishment, and establish classroom norms consistent with that definition? Here too, Lampert stakes out a particular kind of territory and then explains how she works toward the goals he has defined.

In a final theoretical chapter, Lampert presents an elaborated model of teaching practice. There she reframes the problems of teaching multiple students at the same time, and the social complexities of practice; the problems of teaching over time; the complexities of teaching content with a curriculum that is largely problem-based; and the complexities of teaching in an environment where all the actors – students as well as the teacher – are taken seriously as contributors to a goal-oriented, emergent agenda. This model, and the book, raise far more questions than they resolve. But that is as it should be. Lampert has taken an ill-understood domain and portrayed its complexity. She has done so in a structured and theoretical way, which makes that complexity accessible and identifies key dimensions of teaching performance and goals. Now that the framework exists, further work by others should move toward the elaboration of the model and toward practical research questions of teacher development toward the kinds of competencies described in it.

***Ball, Bass, and Colleagues' Study of Mathematical Knowledge for Teaching.***

Deborah Ball, Hyman Bass, and colleagues have embarked on a number of projects (e.g., the Study of Instructional Improvement, the Mathematics Teaching and Learning to Teach Project, the Learning Mathematics for Teaching Project, and the Center for Proficiency in Teaching Mathematics) aimed at understanding the mathematical competencies that underlie teaching. Like the work described above, this growing body of work is predicated on the assumption that mathematics teaching is a deeply mathematical act that is built on a base of mathematical understanding and that also calls for different types of knowledge.

The group's research agenda, writ large, is to understand the mathematical underpinnings for a broad range of pedagogical undertakings, to understand how the teachers' knowledge shapes their classroom practices, and how those practices ultimately affect student learning in mathematics. Papers that describe this agenda and document some progress toward its achievement, include Ball & Bass, 2000, 2003b; Ball & Rowan, 2004; Cohen, Raudenbush, & Ball, 2003; Hill & Ball, 2004; Hill, Schilling, & Ball, 2004; Hill, Rowan, & Ball, in press; the RAND mathematics study panel report, 2002; and the Study of Instructional Improvement, 2002.

A central component of this enterprise is the creation of a series of measures that serve to document teacher knowledge and its impact (see http://www.sii.soe.umich.edu/instruments.html, and Study of Instructional Improvement, 2002). As an example, one of the project's released assessment items shows three hypothetical students' work on multiplying multi-digit numbers:

| Student A | Student B | Student C |
|---|---|---|
| 35<br>x 25<br>125<br>+75<br>875 | 35<br>x25<br>175<br>+700<br>875 | 35<br>2<br>x 5<br>25<br>150<br>100<br>0<br>+6 0<br>875 |

## REFERENCES


1. Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: MIT Press.
2. Warren, B., Ballenger, C., Ogonowski, M., Rosebery, A., & Hudicourt-Barnes, J. (2001). Rethinking diversity in learning science: The logic of everyday sense-making. *Journal of Research in Science Teaching* 38: 1-24.
3. Warren, B. & Rosebery, A. (1995). Equity in the future tense: Redefining relationships among teachers, students, and science in linguistic minority classrooms. In W. Secada, E. Fennema and L. Adajian (Eds.), *New directions for equity in mathematics education* (pp. 298-328). NY: Cambridge University Press.
4. Watson, J. (1930). *Behaviorism (2nd edition).* New York: Norton.
5. Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity.* Cambridge: Cambridge University Press.
6. Wertheimer, Max. (1959). *Productive Thinking*. New York: Harper & Row.

7. What Works Clearinghouse (2004a). Curriculum-based interventions for improving K-12 mathematics achievement – middle school. Downloaded from <http://www.whatworks.ed.gov/>, February 27, 2005.