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POROUS CONTINUOUS MEDIUMS LIQUID SATURATED MOTION WITH ARBITRARY DYNAMIC LOADS

[1]Mukhamadjon Usarovich Musaev, [2]Rakhmankulov Raimkul, [3]Ulugbek Bekmuratov Nurali ugli

Almalik branch of Tashkent state technical university named after I. Karimov Almalik, Uzbekistan [1].,

Almalik branch of Tashkent state technical university named after I. Karimov Almalik city [2]. ,teacher

Almalik branch of Tashkent state technical university named after I. Karimov.

Almalik, Uzbekistan[3]

asadbek50@mail.ru.

ABSTRACT

The article considers porous fluid-saturated continuous media particles motion problem under the arbitrary dynamic loads action. From the point of theory view, this continuous medium is one of the rocks and soft soils models. A general equation is obtained for porous fluid-saturated continuous media motion in three-dimensional space in mass forces absence. It is proved that if an arbitrary dynamic load acts on the medium, then three waves will go in space, two of which are compression (extension) waves, and the third wave is a shear wave.

Keywords: porous solid media saturated with liquid, two-component system, soil movements, solid skeleton, dynamic external load, skeleton stress tensor, strain tensor.

INTRODUCTION

In theory terms, soft soils and rocks are modeled as porous, fluid-saturated continuous media. In general, a theoretical study soil mechanics problem is difficult even if we confine ourselves to civil engineering design tasks. In problems solving of blast waves propagation in the soil, special attention is paid to physical parameters numerical values choice necessary for applying one or another theoretical model of the soil when radiating the laws of motion of the particles of the latter. This is due to the fact that natural soil mechanical behavior under the applied dynamic load influence is very complex, variable, and not fully understood. It follows from this that theoretical studies in soil mechanics are necessarily associated with ideal soils, the physical properties of natural soils approximate properties. Such a path can provide useful information, if, of course, we keep in mind true picture uncertainty of dynamic loads distribution in natural soils. It is known that even under ordinary conditions, natural soils physical properties, measured in the field and in the laboratory, and ideal soils corresponding properties do not coincide in any way. For this reason, in soils and rocks mechanics, discrepancies between theory and practice are inevitable. An example is the adhesion and friction properties, which are some of the main parameters that determine shear deformation. They are determined by significantly different types, interaction forces types between the corresponding continuous medium particles. With this representation, soft soils and rocks are considered, in essence, as a granular structure, but their resistance to deformation also depends on the liquid (gas) content in the pores and on the external conditions imposed during the deformation.

Consistent theories of soil motion, taking into account the consistent fields of stress and velocity, and strain, were first proposed by many researchers. Among such works devoted to this problem, in which various schemes were used with respect to the equation of state, are the studies of S. S. Grigoryan [I]; A.YU. Ishlinsky, N.V. Zvolinsky, N.Z. Stepanenko [2]; Kh.A. Rakhmatullina [4], A.Ya. Sagomonyan [5] and others.

These works are mainly devoted to the study of one-dimensional soil motion with a certain relationship between the average hydrostatic pressure and volumetric deformation or between axial stress and deformation.

A model of the mechanical behavior of the soil, according to which natural soil is presented as a two-component system consisting of a porous elastic body through which fluid can leak, was proposed by V.N. Nikolayevsky [3], Y.N. Frenkel [6], M.A. Bio [7] and others. The theory he proposed was used to study various processes characterized by small deformations.

Problem solution. In this paper, we consider the problem of the motion of particles of porous fluid-saturated continuous media under the action of arbitrary dynamic loads. From the point of view of theory, the aforementioned continuous medium is one of the many models of rocks and soft soils. Suppose that in a certain direction a force acts on an element of a continuous medium, the normal to which is ϑ^r . If the surface area of an element is denoted by, then the acting force on this element will be equal to $T^r dF$. Then, taking the Cartesian coordinate system in space and considering the infinitesimal tetrahedron in this space, it is easy to show that the force laid at a certain point is a linear homogeneous function of the normal ϑ^r , i.e.

$$T^r dF = E^{rs}_{\bullet\bullet}\vartheta^s dF \quad (1)$$

If we assume that the fluid content in the pores of a unit volume is equal to β_0 then the load acting at a certain point in the porous fluid-saturated continuous medium is distributed proportionally to β_0 , i.e.

$$T_1^r = (1-\beta_0)T^r \quad (2)$$

$$T_2^r = \beta_0 T^r \quad (3)$$

where $T_1^r dF_1$ and $T_2^r dF_2$ are the forces acting on the solid skeleton and on the liquid, and in the Cartesian coordinate system they will be linear homogeneous normal functions. Then the forces acting on the solid skeleton and on the liquid in this element will be respectively equal:

$$T_1^r dF_1 = E^{rs}_{1\bullet\bullet}\vartheta_s dF_1 \quad (4)$$

$$T_2^r dF_2 = E^{r}_{2\bullet\bullet}\vartheta_s dF_2 \quad (5)$$

where dF_1 and dF_2 indicate the surfaces occupied by the solid skeleton and the fluid of elementary surface dF , and

$$(1-\beta_0)dF = dF_1 \text{ и } \beta_0 dF = dF_2$$

In expressions (4) and (5), the tensors $E^{rs}_{1\bullet\bullet}$ and $E^{rs}_{2\bullet\bullet}$ are symmetric in r and s, and depends only on the coordinates of the point where the load acts. Moreover, $E^{rs}_{2\bullet\bullet}$ does not depend on the direction of the normal ϑ_s . This follows from the law of Archimedes, i.e. in liquids, the stresses on the element are always normal to this element $E^{rs}_{1\bullet\bullet}$ is a contravariant tensor of the second rank and we call it the skeleton stress tensor Let a volume $d\tau$ inside a porous, fluid-saturated continuous medium ((from which $\beta_0\tau$ - takes fluid; $(1-\beta_0)d\tau$ – takes solid particles of the skeleton) be bounded by dF surface. Then the condition of dynamic equilibrium of a given volume under the action of a dynamic external load has the form:

$$\iint T_1^r \lambda_r^{(1)} dF_1 + \iiint [\rho_{11}(Q_1^r - f_1^r) + \rho_{12}(Q_2^r - f_2^r)]\lambda_r^{(1)}.d\tau_1 = 0$$

$$\iint T_2^r \lambda_r^{(2)} dF_2 + \iiint [\rho_{12}(Q_1^r - f_1^r) + \rho_{22}(Q_2^r - f_2^r)]\lambda_r^{(2)}.d\tau_2 = 0 \qquad (6)$$

where f_1^r и f_2^r are acceleration vectors of solid and liquid particles

$d\tau_1$ и $d\tau_2$ are corresponding elementary volumes,

$\lambda_r^{(1)}$ и $\lambda_r^{(2)}$ are corresponding arbitrary constants.

If ϑ_s is unit normal vector to the surface element, directed to the outside, then according to (4) and (5) we get $\iint E_{1\bullet\bullet}^{rs} \lambda_r^{(1)} \vartheta_s dF_1 + \iiint [\rho_{11}(Q_1^r - f_1^r) + \rho_{12}(Q_2^r - f_2^r)]\lambda_r^{(1)} d\tau_1 = 0$

$$\iint E_{2\bullet\bullet}^{rs} \lambda_r^{(2)} \vartheta_s dF_2 + \iiint [\rho_{12}(Q_1^r - f_1^r) + \rho_{22}(Q_2^r - f_2^r)]\lambda_r^{(2)} d\tau_2 = 0 \qquad (7)$$

Using Green's theorem and taking into account that $\lambda_{r,s} = 0$, we reduce system (7) to the form:

$$\iiint [\rho_{11}(Q_1^r - f_1^r) + \rho_{12}(Q_2^r - f_2^r) + E_{(1),\ldots,S}^{rs}]\lambda_r^{(1)} d\tau_1 = 0$$

$$\iiint [\rho_{12}(Q_1^r - f_1^r) + \rho_{22}(Q_2^r - f_2^r) + E_{(2),\ldots,S}^{rs}]\lambda_r^{(2)} d\tau_2 = 0 \qquad (8)$$

Moreover, these relations are valid for any volume and any parallel λ_r vector field. Therefore, at each point the equations are valid:

$$E_{(1),\ldots,S}^{rs} + [\rho_{11}(Q_1^r - f_1^r) + \rho_{12}(Q_2^r - f_2^r)] = 0$$

$$E_{(2),\ldots,S}^{rs} + [\rho_{12}(Q_1^r - f_1^r) + \rho_{22}(Q_2^r - f_2^r)] = 0 \qquad (9)$$

moreover, between the mass coefficients of the skeleton ρ_T and fluid $\rho_{Ж}$ there are relations:

$$\rho_{11} + \rho_{12} = (1 - \beta_0)\rho_T$$

$$\rho_{12} + \rho_{22} = \beta_0 \rho_{Ж} \qquad (10)$$

where ρ_T is the density of the skeleton; $\rho_{Ж}$ is the density of the filling fluid.

System (9) is the equation of motion of particles in a porous, fluid-saturated continuous medium in tensor form. In the absence of mass forces Q_1^r and this system has the form:

$$E_{(1),\ldots,S}^{rs} = \rho_{11} f_1^r + \rho_{12} f_1^r$$

$$E_{(2),\ldots,S}^{rs} = \rho_{12} f_2^r + \rho_{22} f_2^r \qquad (11)$$

equations (11) can be written in covariant form:

$$q^{sl} E_{(1)rs,1} = \rho_{11} f_{(1)r} + \rho_{12} f_{(1)r}$$

$$q^{sl} E_{(2)rs,1} = \rho_{12} f_{(2)r} + \rho_{22} f_{(2)r} \qquad (12)$$

Considering the properties of the tensor $E_{(2)}$, т.е. $E_{(2)rs} = \sigma q_{rs}^{(1)}$, where $\sigma -$ a is the scalar and opposite in sign pressure and $q^{sl}E_{(2)rs,l} = q^{sl}(\sigma q_{rs})_{,t} = q^{sl}\frac{\partial\sigma}{\partial x^r}q_{rs} = \frac{\partial\sigma}{\partial x^r}$ (13) system (12) we bring to the form: $q^{sl}E_{(1)rs,l} = \rho_{11}f_{(1)r} + \rho_{12}f_{(2)r}$ $\frac{\partial\sigma}{\partial x^r} = \rho_{12}f_{(1)r} + \rho_{22}f_{(2)r}$ (14)

Suppose that under the action of an external load, the particles of the skeleton and the liquid, which are located at x^r point, before the action of an external load, respectively move to $\overrightarrow{u_r}$ и $\overrightarrow{U_r}$ distance. Then the acceleration vectors of these particles will be:

$$f_{(1)r} = \frac{\partial\overrightarrow{u_r}}{\partial t^2}; \qquad f_{(2)r} = \frac{\partial\overrightarrow{U_r}}{\partial t^2} \quad (15)$$

and system (14) will have the form:

$$q^{sl}E_{(1),rs,l} = \rho_{11}\frac{\partial^2\overrightarrow{u}_r}{\partial t^2} + \rho_{12}\frac{\partial^2\overrightarrow{U}_r}{\partial t^2},$$

$$\frac{\partial\sigma}{\partial r} = \rho_{12}\frac{\partial^2\overrightarrow{u}_r}{\partial t^2} + \rho_{22}\frac{\partial^2\overrightarrow{U}_r}{\partial t^2} \qquad (16)$$

If the dependence of the stress tensor of the skeleton on the strain tensor is taken in the form:

$$E_{(1)rS} = C_{rs}^{mn}e_{mn} + Q\frac{\partial U_r}{\partial x^r} \quad (17)$$

where C_{rs}^{mn} - depend only on the coordinates and form a mixed fourth-rank tensor, called the elastic modulus tensor. It is also clear that, without loss of generality, this tensor can be chosen symmetric in both subscripts and superscripts.

Replacing the tensor $E_{(1)rS}$ in (16) with its expression (17), we obtain:

$$q^{sl}C_{rs}^{mn}e_{mn,l} + q^{sl}C_{rs,l}^{mn}e_{mn} + Q\frac{\partial U_r}{\partial x^r} = \rho_{11}\frac{\partial^2\overrightarrow{u}}{\partial t^2} + \rho_{12}\frac{\partial\overrightarrow{U_r}}{\partial t^2}$$

$$\frac{\partial\sigma}{\partial x^r} = \rho_{12}\frac{\partial\overrightarrow{u_r}}{\partial t^2} + \rho_{22}\frac{\partial\overrightarrow{U_r}}{\partial t^2} \quad (18)$$

This is the general equation for the motion of porous fluid-saturated continuous media in three-dimensional space in the absence of mass forces. If the continuous medium is homogeneous, then the same deformation at different points causes the same stress. Well then $E_{(1)rS,l} = 0$,если $e_{rs,l} = 0$. Therefore, in order for the medium to be elastically homogeneous, the following conditions are necessary and sufficient:

$$C_{rs,;}^{mn} = 0 \qquad (19)$$

Let us now consider the case when a porous, fluid-saturated continuous medium is homogeneous and isotropic, i.e. when the strain-strain dependence has the following form:

$$E_{(1)rs} = \theta q_{rs} + 2\mu e_{rs} + Q\varepsilon q_{rs}$$

$$\sigma = Q\theta + R\varepsilon \qquad (20)$$

where λ, μ, Q и R - elastic parameters of the medium,

θ и ε – volumetric expansion of the skeleton and fluid, i.e.

$$\theta = \vec{U}_r^{\,r} = q^{mn}.e_{mn} \quad \varepsilon = \vec{U}_r^{\,r} = q^{mn}.\varepsilon_{mn} \quad (21)$$

The relation (20) and (21) shows that for an isotropic medium the modules are connected

$$C_{rs}^{mn} = q^{mn}q_{rs} + \mu(\delta_r^m\delta_s^n + \delta_r^n\delta_s^m) \qquad (22)$$

Then the stress tensor of the skeleton will take the form:

$$q^{st}E_{(1)rs,l} = \frac{\partial\theta}{\partial x^r} + 2\mu q^{sl}e_{rs,l} + Q\frac{\partial\varepsilon}{\partial x^r}$$

where
$$q^{sl}E_{(1)rs,l} = \frac{1}{2}q^{sl}(\vec{U}_{r,sl} + \vec{U}_{s,rl}) = \frac{1}{2}q^{sl}\vec{U}_{r,sl} + \frac{1}{2}\frac{\partial\theta}{\partial x^r}$$

Given these relationships, the equations of motion of elastically porous fluid-saturated media, when they are homogeneous isotropic, will have the following form:

$$(\lambda+\mu)\frac{\partial\theta}{\partial x^r} + \mu q^{sl}\vec{U}_{r,sl} + Q\frac{\partial\varepsilon}{\partial x^r} = \rho_{11}\frac{\partial^2\vec{u_r}}{\partial t^2} + \rho_{12}\frac{\partial^2\vec{U}_r}{\partial t^2}$$

$$\frac{\partial\sigma}{\partial x^r} = \rho_{12}\frac{\partial^2\vec{u}_r}{\partial t^2} + \rho_{22}\frac{\partial^2\vec{U}_r}{\partial t^2} \qquad (23)$$

where $\vec{u} = u\vec{i} + v\vec{j} + w\vec{k}$ - skeleton particle displacement vector,

$\vec{U} = U\vec{i} + V\vec{j} + W\vec{k}$ - fluid displacement vector. In the components of equation (23) has the form:

$$(\lambda+\mu)\frac{\partial\theta}{\partial x} + \mu\nabla u + Q\frac{\partial\varepsilon}{\partial x} = \rho_{11}\frac{\partial^2 u}{\partial t^2} + \rho_{12}\frac{\partial^2 U}{\partial t^2}$$

$$(\lambda+\mu)\frac{\partial\theta}{\partial y} + \mu\nabla v + Q\frac{\partial\varepsilon}{\partial y} = \rho_{11}\frac{\partial^2 v}{\partial t^2} + \rho_{12}\frac{\partial^2 V}{\partial t^2}$$

$$(\lambda+\mu)\frac{\partial\theta}{\partial z} + \mu\nabla w + Q\frac{\partial\varepsilon}{\partial z} = \rho_{11}\frac{\partial^2 w}{\partial t^2} + \rho_{12}\frac{\partial^2 W}{\partial t^2}$$

$$Q\frac{\partial\theta}{\partial x} + R\frac{\partial\varepsilon}{\partial x} = \rho_{11}\frac{\partial^2 u}{\partial t^2} + \rho_{12}\frac{\partial^2 U}{\partial t^2} \qquad (24)$$

$$Q\frac{\partial\theta}{\partial y} + R\frac{\partial\varepsilon}{\partial y} = \rho_{11}\frac{\partial^2 v}{\partial t^2} + \rho_{12}\frac{\partial^2 V}{\partial t^2}$$

$$Q\frac{\partial\theta}{\partial z} + R\frac{\partial\varepsilon}{\partial z} = \rho_{11}\frac{\partial^2 w}{\partial t^2} + \rho_{12}\frac{\partial^2 W}{\partial t^2}$$

After some "transformed" this system we bring it to the form:

$$(\lambda+2\mu)\nabla\theta + Q\nabla\varepsilon = \rho_{11}\frac{\partial^2\theta}{\partial t^2} + \rho_{12}\frac{\partial^2\varepsilon}{\partial t^2}$$

$$Q\nabla\theta + R\nabla\varepsilon = \frac{\partial^2\theta}{\partial t^2} + \rho_{12}\frac{\partial^2\varepsilon}{\partial t^2} \qquad (25)$$

$$\mu\nabla(rot\vec{u}) = \rho_3 \frac{\partial^2}{\partial t^2}(rot\vec{u}) \quad , \quad \frac{\partial^2}{\partial t^2}(rot\vec{U}) = -\frac{\rho_{12}}{\rho_{22}} \cdot \frac{\partial^2}{\partial t^2}(rot\vec{u})$$

where $\rho_3 = (1-\beta_0)\rho_T - \dfrac{\beta_0\rho_Ж\rho_{12}}{\beta_0\rho_Ж - \rho_{12}}$, ∇ - Lapas operator

Taking into account (2), (3), (4), and (5), system (11) is reduced to the form:

$$E_{\bullet\bullet,s} = (\rho_{11}+\rho_{12}) f_1^r + (\rho_{12}+\rho_{22}) f_2^r$$

$$(1-2\beta_0)E^{sr}_{\bullet\bullet,s} = (\rho_{11}+\rho_{12}) f_1^r + (\rho_{12}+\rho_{22}) f_2^r \quad (26)$$

If we assume that the liquid content of the total volume of the continuous medium is less than 50% (otherwise, a continuous medium with a content of 50% or more of the liquid will acquire creep properties and we will not consider such continuous media) then from (26) we obtain the following: $f_2^r = kf_1^r$ (27), where k is the coefficient of proportionality and is determined through other environmental parameters of the following expression:

$$k = \frac{\beta_0(1-\beta_0)\rho_T - \rho_{12}}{\beta_0(1-\beta_0)\rho_Ж - \rho_{12}} \quad (27)$$

So in porous, liquid-saturated homogeneous continuous media between the acceleration vectors of solid and liquid particles, there is a law of proportionality, which is written in the form:

$$\frac{\partial^2\vec{U}}{\partial t^2} = k\frac{\partial\vec{u}}{\partial t^2}; \quad (28)$$

In the projection, expression (27) has the form:

$$\frac{\partial^2 U}{\partial t^2} = k\frac{\partial^2 u}{\partial t^2}; \quad \frac{\partial^2 V}{\partial t^2} = k\frac{\partial^2 v}{\partial t^2}; \quad \frac{\partial^2 W}{\partial t^2} = k\frac{\partial^2 w}{\partial t^2}. \quad (29)$$

Then it obviously follows from (29) that

$$\frac{\partial^2\varepsilon}{\partial t^2} = k\frac{\partial\theta}{\partial t^2} \quad (30)$$

Given these relationships, the equation of motion (25) is reduced to three wave equations:

$$\nabla^2\theta = \frac{1}{a_1^2}\frac{\partial^2\theta}{\partial t^2}; \quad \nabla^2\varepsilon = \frac{1}{a_2^2}\frac{\partial^2\varepsilon}{\partial t^2}; \quad \nabla(rot\vec{U}) = \frac{1}{b}\frac{\partial^2}{\partial t^2}(rot\vec{u}). \quad (31)$$

где $a_1 = [\lambda + 2\mu - \frac{Q^2}{R})/\rho_1]^{\frac{1}{2}}$; $a_2 = [(\lambda + 2\mu - \frac{Q^2}{R})/\rho_2]^{\frac{1}{2}}$; $b = (\frac{\mu}{\rho_3})^{\frac{1}{2}}$ (32)

$$\rho_1 = (1-\beta_0)\rho_T - k\beta_0\frac{Q}{R}\rho_Ж + (k+1)(1+\frac{Q}{R})\rho_{12};$$

$$\rho_2 = \frac{1}{R}[(\lambda+2\mu)\beta_0\rho_Ж - \frac{(1-\beta_0)}{k} \bullet Q\rho_T - (1-\frac{1}{k})(\lambda+2\mu+Q)\rho_{12}] \quad (33)$$

In conclusion, we note that if a porous fluid-saturated continuous medium fills, three-dimensional euclidean space and an arbitrary dynamic load act on it, in this case, three types of waves propagate in three-dimensional space, which are pure compression (extension) waves traveling at speeds a_1 and a_2, respectively, and the third wave traveling at a speed b is a shear wave. The velocities of these waves are determined respectively from expressions (32).Obviously, for a uniformly elastic medium under certain initial and boundary conditions, the solution of system (30) does not present any difficulty. So, a porous particles motion, fluid-saturated elastic continuous medium filling a three-dimensional space under dynamic loads acting on them is described by a system of six partial differential equations, which in turn is reduced to three wave equations with three different velocities.

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