NUMERICAL METHODS OF BRAILOVSKY AND ALLEN-CHEN FOR SOLVING THE BURGERS EQUATION

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ABSTRACT

It is proposed to use the Burgers equation as a model equation in the development of a technique for the numerical study of the convergence and stability parameters of difference schemes when calculating the dynamics of a viscous medium by the method of grids. Exact solutions of the above equation are presented. To solve the parabolic burgers equation, two-step finite-difference methods are used.

Key words: Two-step method, explicit scheme, finite difference scheme, predictor, corrector, viscous term, first order of accuracy.

INTRODUCTION

This article describes and studies in detail the finite-difference schemes of Brailovskiy and Alain-Chen, which can be used to solve the simplest model equations [1]. We restrict ourselves to considering the parabolic Burgers equation in partial derivatives. Consider an explicit two-step difference schemes having the first order of accuracy. Methods of Brailovsky and Alena-Chen are studied in detail [2]

The Burgers equation is a parabolic partial differential equation. It is described as follows [3].

$$\frac{\partial U}{\partial t} + c\frac{\partial U}{\partial x} = \mu \frac{\partial^2 U}{\partial x^2}, \ \mu > 0 \qquad (1)$$

Here analogs of physical quantities and functions of the process of dynamics of viscous media are: c - flow rate; t is time; x - coordinate along the stream; μ - viscosity [6].

We want to construct solutions of the Burgers equation $x \in [0, L]$, with $t \in [0, \infty]$, the boundary conditions

$$U[0,t] = U[L,t] = 0, \ \ t \in [0, \infty], \qquad (2)$$

At = μ =0, the wave equation is obtained, and at c = 0 the heat equation.

Equations (1) can be written in the divergent form

$$U_t + \overline{F} = 0 \qquad (2)$$

where $\overline{F}$ is determined by the ratio

$$\overline{F} = cU + bU^2/2 - \mu U_x \qquad (3)$$

Equations (1) can be written in another way

$$U_t + F_x = \mu U_{xx} \qquad (4)$$

where

$$F = cU + bU^2/2 \qquad (5)$$

DIGITAL METHOD

Application of finite difference methods for solving the Burgers equation.

1. Brailovsky's method. I.Yu.Brailovskaya proposed the following scheme for solving the equation [1].

$$U_t + F_x = \mu U_{xx}$$

Predictor

$$U_j^{\overline{n+1}} = U_j^n - \frac{\Delta t}{2\Delta x}(F_{j+1}^n - F_{j-1}^n) + r(U_{j+1}^n - 2U_j^n + U_{j-1}^n) \quad (6)$$

Corrector

$$U_j^{n+1} = U_j^n - \frac{\Delta t}{2\Delta x}(F_{j+1}^{\overline{n+1}} - F_{j-1}^{\overline{n+1}}) + r(U_{j+1}^n - 2U_j^n + U_{j-1}^n)$$

where

$$F_{j+1}^n = cU_{j+1}^n + bU_{j+1}^n / 2 \quad r = \frac{\mu \Delta t}{\Delta x} \quad (7)$$

2. Alain-Chen method. This diagram is as follows.

Predictor

$$U_j^{\overline{n+1}} = U_j^n - \frac{\Delta t}{2\Delta x}(F_{j+1}^n - F_{j-1}^n) + r(U_{j+1}^n - 2U_j^{\overline{n+1}} + U_{j-1}^n) \quad (8)$$

Corrector

$$U_j^{n+1} = U_j^n - \frac{\Delta t}{2\Delta x}(F_{j+1}^{\overline{n+1}} - F_{j-1}^{\overline{n+1}}) + r(U_{j+1}^{\overline{n+1}} - 2U_j^{n+1} + U_{j-1}^{\overline{n+1}})$$

CALCULATION RESULTS

Here are some specific examples to illustrate the numerical methods briefly described above. The calculation results are compared with the calculations [1].

1. Brailovsky's method.

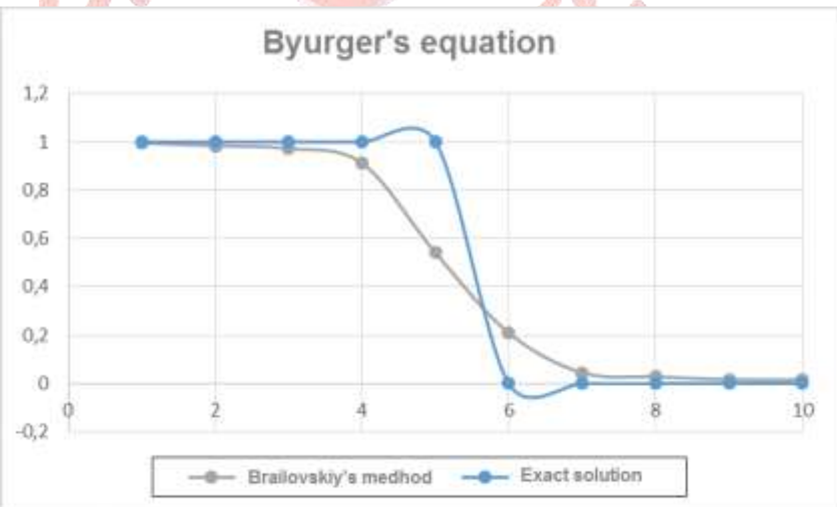

Figure1.

2. Alain-Chen method.

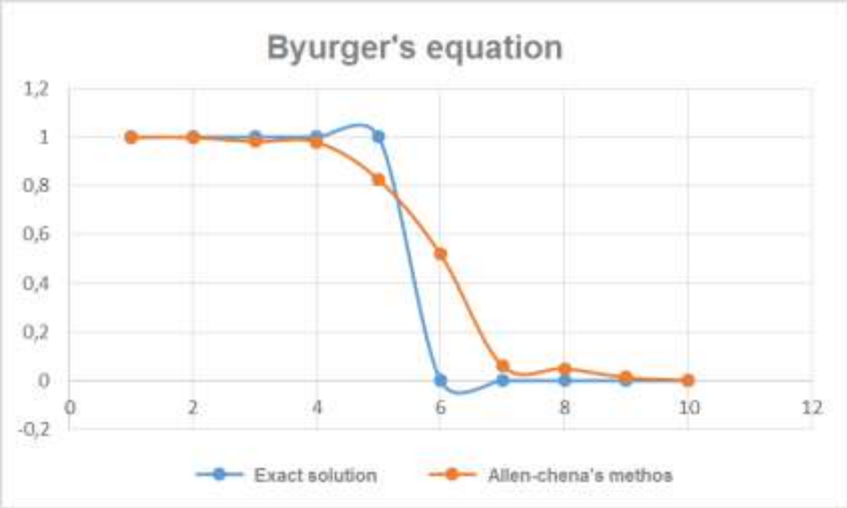

Figure 2.

CONCLUSION

Comparison of calculation results is carried out. It is shown that these finite-difference schemes give very close computational results for the exact solution of the Burgers equation. Explicit first-order one-step schemes give satisfactory agreement between the calculated data. Numerical calculations make it possible not only to discover new effects that arise when solving the Burgers equation on a finite interval.

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