THE NECESSERY CONDITIONS OF OPTIMALITY IN THE MINIMAX CONTROL PROBLEM FOR DIFFERENTIAL INCLUSION

[1]Otakulov Salim, [2]Kholiyarova Feruza Khafizovna
Doctor of physical and mathematical Sciences, Professor,Samarkand branch of Tashkent University of information technologies, Samarkand, Uzbekistan[1], Teacher, Samarkand branch of Tashkent University of information technologies, Samarkand, Uzbekistan[2]
otakulov52@mail.ru,, feruza1377@mail.ru

ABSTRACT:

In the paper we considered one class controlling differential inclusions with delay. For the control system the minimax problem is researched. In the nosmooth control problem the necessary conditions of optimality are obtained.

Keywords: differential inclusions, control system, nosmooth functional, minimax problem, conditions of optimality.

INTRODUCTION

The theory of differential inclusions has effective applications in the theory of optimal control, in the theory of differential equations with discontinuous right-hand sides, in differential games, in mathematical economics and in other areas of mathematical research [1,2,3].

In studies of control systems under informational constraints, the properties of the ensemble of trajectories, methods for estimating the reachability set and forecasting the phase state of the system, minimax synthesis problems and others are of great interest. One of the approaches used in making decisions in conditions of incomplete information is the principle of guaranteed management. This principle leads to minimax control problems for the ensemble of trajectories of a dynamical system. Various problems of controlling the ensemble of trajectories were considered in [4, 5, 6].

One of the important models of real control processes are systems with delays (deviations). Control problems for such systems are more complex than systems without delay. Moreover, the specific of the constructions method of optimal control depends on the degree to which the delay factor is taken into account in the dynamics of the control system.

In connection with the control problems of dynamical systems with delays under conditions of uncertainty, arise problems of controlling ensembles of trajectories of differential inclusion with delay. Some properties of controlled differential inclusions with delays were studied in work [5, 8, 9]. In particular, the conditions of compactness and convexity of the set of absolutely continuous solutions are revealed. In [9], for such systems, the control problem with terminal constraints was considered.

2. The object of study and methods. Consider a mathematical model of a control system that is described by a controlled differential inclusion of the form

$$\frac{dx}{dt} \in A(t)x + \sum_{i=1}^{k} A_i(t)x(t-h_i) + b(t,u), t \ge t_0, \ u \in V, \qquad (1)$$

where x is the state n-vector, u is the control m-vector, $A(t)$ and $A_i(t)$, $i=\overline{1,n}$, – $n \times n$ -matrices, $b(t,u)$ is a nonempty compact from R^n, $h_i > 0$, $i=\overline{1,n}$, - are delay constants, V is a convex compact from R^m, R^n is n-dimensional Euclidean space with the scalar product $(x,y)=\sum_{i-1}^{n} x_i y_i$ and the norm

$\|x\| = \sqrt{(x,x)}$. We assume that the right-hand side of the differential inclusion (1) satisfy the following conditions:

1) the elements of the matrices $A(t)$ and $A_i(t), i = \overline{1,n}$, summable on $T = [t_0, t_1]$;

2) a multivalued mapping $(t,v) \to b(t,v)$ is measurable in a variable $t \in T$ and continuous in a variable $v \in V$, moreover $\underset{\gamma \in b(t,u)}{Sup} \|\gamma\| \le \beta(t), \ \forall (t,v) \in T \times V$, where $\beta(t)$ is a summable function on $T = [t_0, t_1]$.

As admissible controls for system (1), we choose measurable bounded functions $u = u(t), t \in T$, that take values almost everywhere on T from compact V. Denote by $U(T)$ - the set of all admissible controls. Let $C^n(T_0)$ be the space of n -vector functions continuous on $T_0 = [t_0 - h, t_0]$, where $h = \max_{i=\overline{1,k}} h_i$. An admissible trajectory corresponding to the control $u(\cdot) \in U(T)$ and the initial function $\varphi_0(\cdot) \in C^n(T_0)$ is called continuous on $T_1 = [t_0 - h, t_1]$ and absolutely continuous on T an n -vector function $x = x(t)$, satisfying differential inclusion (1) and the initial condition $x(t) = \varphi_0(t), t \in T_0$. Denote by $H(u, \varphi_0)$ the set of all admissible trajectories corresponding to the admissible control $u(\cdot)$ and the initial function $\varphi_0(\cdot) \in C^n(T_0)$.

Consider the set $X(t_1, u, \varphi_0) = \{\xi \in R^n : \xi = x(t_1), \ x(\cdot) \in H(u, \varphi_0)\}, t \in T$. According to the results of [8], when the above conditions are met, the set $X(t_1, u, \varphi_0)$ is convex, closed and bounded for any $u(\cdot) \in U(T)$ and $\varphi_0(\cdot) \in C^n(T_0)$.

Let the quality of the control process of system (1) be evaluated by a nonsmooth terminal functional $J(x(\cdot)) = g(x(t_1)), \ \ x(\cdot) \in H(u, \varphi_0)$, where $g(x) = \sum_{i=1}^{k} \min_{z_i \in Z_i} (z_i, P_i x)$, P_i is a $m_i \times n$ -matrix, Z_i is a closed bounded set from R^{m_i}. The purpose of management is to obtain the guaranteed value of the criterion $J(x(\cdot)) = g(x(t_1)), \ \ x(\cdot) \in H(u, \varphi_0)$. According to this goal, we consider the problem of minimizing the functional

$$\Phi(u) \equiv \underset{x(\cdot) \in H(u,\varphi_0)}{Sup} J(x(\cdot)) = \max_{\xi \in X(t_1, u, \varphi_0)} g(\xi), \tag{2}$$

it is required to find a control $u^*(\cdot) \in U(T)$, satisfying the condition

$$\min_{u \in U(T)} \max \{g(\xi) : \xi \in X(t_1, u, \varphi_0)\} = \max \{g(\xi) : \xi \in X(t_1, u^*, \varphi_0)\}.$$

So, the following minimax optimal control problem for the ensemble of trajectories of system (1) has been posed:

$$\max_{\xi \in X(t_1, u, \varphi_0)} g(\xi) \to \min, \ u \in U(T). \tag{3}$$

3. The main results. We study necessary optimality conditions in problem (3). We put: $y = \sum_{i=1}^{k} y_i, \ y_i = P_i' z_i \in R^n, \ Y = \sum_{i=1}^{k} Y_i, Y_i = P_i' Z_i, \ coY = \sum_{i=1}^{k} coY_i$ is the convex hull of the set Y. Using the minimax theorem from convex analysis, we have:

$\Phi(u)=\max\limits_{\xi\in X(t_1,u,\varphi_0)}\min\limits_{y\in coY}(\xi,y)=\min\limits_{y\in coY}C(X(t_i,u,\varphi_0),y)$, where $C(X(t_1,u,\varphi_0),\psi)=\max\limits_{\xi\in X(t_1,u,\varphi_0)}(\xi,\psi)$ is the support function of a convex compact set $X(t_1,u,\varphi_0)$.

The following representation is true:

$$X(t_1,u,\varphi_0)=S(\varphi_0)+\int_{t_0}^{t_1}F(t_1,\tau)b(\tau,u(\tau))d\tau,$$

where $F(t,\tau)-n\times n$ is a matrix function satisfying the equation:

$$\frac{\partial F(t,\tau)}{\partial\tau}=-F(t,\tau)A(\tau)-\sum_{i=1}^{k}F(t,\tau+h_i)A_i(\tau+h_i),\ \ \tau\le t,\ \ F(t,t-0)=E,$$

$$F(t,\tau)\equiv 0,\ \ \tau\ge t+0,\ S(\varphi_0)=F(t_1,t_0)\varphi_0(t_0)+\sum_{i=1}^{k}\int_{t_0}^{t_0+h_i}F(t_1,t)A_i(t)\varphi_0(t-h_i)dt.$$

The support function of a set $X(t_1,u,\varphi_0)$ is expressed by the equality

$$C(X(t_1,u,\varphi_0),\psi)=(S(\varphi_0),\psi)+\int_{t_0}^{t_1}C(F(t_1,t)b(t,u(t)),\psi)dt\ .$$

Therefore, for functional (3), the following formula is true:

$$\Phi(u)=\min_{y\in coY}\left[(S(\varphi_0),y)+\int_{t_0}^{t_1}C(F(t_1,t)b(t,u(t)),y)dt\right],\qquad(4)$$

We introduce the functional

$$\rho(y,u)=(S(\varphi_0),y)+\int_{t_0}^{t_1}C(F(t_1,t)b(t,u(t)),y)dt.\qquad(5)$$

Then formula (4) can be written as: $\Phi(u)=\min\limits_{y\in coY}\rho(y,u)$. This representation of functional (5) allows minimax problem (3) to be written in the following form:

$$\Phi(u)\equiv\min_{y\in coY}\rho(y,u)\to\min,\ u\in U(T).\qquad(6)$$

Let $u^*(t),t\in T,$ be the optimal control in problem (3). Then, by virtue of (6), we have: $\min\limits_{y\in coY}\rho(z,u^*)\le\min\limits_{y\in coY}\rho(z,u),\ \forall\, u\in U(T).$ Hence,

$$\rho(y^*,u^*)=\min_{u\in U(T)}\rho(y^*,u),\qquad(7)$$

where $y^*\in coY$ is the global minimum point of the function $y\to\rho(y,u^*),\ y\in coY$. Account the functional (5), from (7) we obtain

$$\int_{t_0}^{t_1}\left[\min_{\vartheta\in V}C(F(t_1,t)b(t,\vartheta),y^*)-C(F(t_1,t)b(t,u^*(t)),y^*)\right]dt=0.$$

Hence, by virtue of the properties of the Lebegues integral, it follows that

$$C(F(t_1,t)b(t,u^*(t)),y^*)=\min_{\upsilon\in V}C(F(t_1,t)b(t,\vartheta),y^*)\qquad(8)$$

for almost all $t\in T$. So, the following necessary optimality condition holds.

Theorem 1. Let $u^* = u^*(t), t \in T$, be the optimal control in problem (3) and $y^* \in coY$ be an voluntary point of the global minimum of the function $y \to \rho(y, u^*)$. Then equality (8) holds for almost all $t \in T$.

Now we give a statement, clarifying the result of Theorem 1.

Consider the function $\mu(y) = (S(\varphi_0), y) + \int_{t_0}^{t_1} \min_{\vartheta \in V} C(F(t_1, t)b(t, \vartheta), y)dt$.

Theorem 2. Let $u^*(t), t \in T$, be the optimal control in problem (3), and the functional $\rho(y, u)$ has the form (5). Then each point of the global minimum of the function $y \to \rho(y, u^*)$ on the set coY is a point of the global minimum of the function $\mu(y)$ on the same set coY.

CONCLUSION

The final result of the study of minimax control problems substantially depends on the properties of the optimized functional. The function $g(x) = \sum_{i=1}^{k} \min_{z_i \in Z_i}(z_i, P_i x)$ used as a terminal functional for estimating the final state of the ensemble of trajectories of controlled differential inclusion (1) has the concavity property.

A new representation $\Phi(u) = \min_{y \in coY} \rho(y, u)$ of the quality control criterion for the ensemble of trajectories made it possible to reduce the minimax problem (3) to the repeated minimization problem of the form (6).

This functional $\rho(y, u)$ of the form (5) is convex in a variable $y \in coY$. Under the additional condition of convexity of the support function $C(b(t, v), \psi)$ in a variable $v \in V$, the functional $\rho(y, u)$ is be convex in the variable $u \in U(T)$.

Here, as a remark, we note that if the multivalued mapping $(t, v) \to b(t, v)$ is continuous in each variable, and the class of admissible controls is limited to piecewise continuous functions, then the minimum condition (8) holds for all $t \in T$.

In this paper, we consider one minimax problem for one class of controlled differential inclusions. For this problem the necessary conditions optimality are studied.

REFERENCES

1. Blagodatskikh V.I, Filippov A.F. Differential inclusions and optimal control, Trudy Mat. Inst, AN SSSR, vol.169 (1985). pp. 194-252.
2. Clarke F. Optimization and nonsmooth analysis, Willey & Sons, Ney York (1983).
3. Polovinkin E.S. Set-valued analysis and differential inclusions, Fizmatlit, Moskow (2014).
4. Plotnikov A.B. The control problem by bunch of trajectories, Sibir. Math. Jurn., vol. 33, № 2 (1992). pp. 196-199.
5. Otakulov S. On the minimization problem of reachable set estimation of control system. IFAC Workshop on Generalized Solution in Control Problems (GSCP-2004). Pereslavl-Zalessky, Russia (September 22-26, 2004). pp. 212-217.
6. Plotnikov A.V., Komleva T.A. Piecewise constant controlled linear fuzzy differential inclusions. Universal Journal of Applied Mathematics. vol.1, №2 (2013). pp. 39-43.

7. Otakulov S. The control problem by ensemble of trajectories for differential inclusions, Lambert Academic Publishing, Riga (2019).
8. Otakulov S., Kholiyarova F.Kh. On the theory controlled differential inclusionswith delay argument. Dokl AN RYz., № 3(2005). pp. 14-17.
9. Otakulov S., Kholiyarova F.Kh. About the control problem an ensemble trajectories of system with delay and under information limited. Abstracts of the international scientific conference "Actual problems of applied mathematics and information technologies" (November 14-15, 2019), National University of Uzbekistan, Tashkent (2019). pp. 169-170.