ORDINARY DIFFERENTIAL EQUATION WITH PARAMETER

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ABSTRACT

An ordinary differential equation with a parameter is considered, a necessary and sufficient condition for the existence of solutions that is independent of the parameter is found. If some relation is satisfied, the equation has a unique solution, otherwise the variety of solutions contains one arbitrary constant,

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Ordinary differential equation with parameter

Consider the equation

$$u''(x) + a(x,t)u'(x) + в(x,t)u(x) = c(x,t) \qquad (1)$$

Where $a, b, c \in C^3$

Since a nonzero solution is found that does not depend on t, then instead of (1), the relation

$$a_t u' + в_t u = c_t \qquad (2)$$

Further it will be essential which of the three coefficients, a, b, c are independent of t.

1.Let$a_t \equiv 0$.Then it follows from (2) that the quantity

$$q(x) = c_t : b_t \qquad (3)$$

independent of t and coincides with the solution u (x).

Integrating (3) over t, we got:

$$c(x.t) = q(x)в(x.t) + p(x). \qquad (4)$$

Substituting y = q (x) into (1), we have

$$q''(x) + a(x)q'(x) - p(x) \equiv 0 \qquad (5)$$

2. Let now a_$a_t \neq 0$.In this case, the system of equations (1) and (2) is obviously equivalent to the following two

$$u' = -\frac{b_t}{a_t}u + \frac{c_t}{a_t} \quad (6)$$

$$u'' = \left(-b + a\frac{b_t}{a_t}\right)u + c - a\frac{c_t}{a_t} \quad (7).$$

Differentiating relation (6) and (7) with respect to t, we obtain:

$$\frac{\partial}{\partial t}\left(\frac{b_t}{a_t}\right)u_t = \frac{\partial}{\partial t}\left(\frac{c_t}{a_t}\right) \quad (8)$$

If $b\_\frac{b_t}{a_t} \neq 0$, then u(x)=q(x) where q (x) denotes the ratio of the coefficients from (8) and it should not depend on t. After the corresponding integrations, we have

$$c(x.t) = p(x)a(x,t) + q(x)b(x,t) + R(x) \quad (9)$$

Where$p(x), q(x), R(x)$—famous functions, substituting$y$ =q(x) and (9) in (6) and (7) m we get $p(x) \equiv \dot{q}(x), \ddot{q}(x) \equiv R(x)$. (10) If$\frac{b_t}{a_t} \neq 0$, then from(8) followed by the necessary conditio$\frac{c_m}{a_m} \equiv 0$denote:

$$\frac{b_t}{a_t} = p(x), \quad \frac{c_t}{a_t} = r(x) \quad (11)$$

Integrating relations over t, we have

$$b(x.t) = p(x)\, a(x,t) + \kappa(x) \quad (12)$$

$$c(x,t) = r(x)a(x,t) + p(x)$$

where p (x), k (x), h (x), p (x) are known functions, substituting (11), (12) in (6) and (7) we obtain

$$u' = -p(x)u + ч(x) \quad (13)$$

$$u'' = -k(x)н + p(x) \quad (14)$$

Differentiating (13) and equating (14), we obtain

$$A(x)u(x) = b(x) \quad (15)$$

$$A(x) = p''(x) - p'(x) + q(x) \quad (16)$$

$$B(x) = p(x) - r'(x) + p(x)r(x)$$

If A ≠ 0, then flat

q (x) = B: A and substituting y = q (x) in (13) and (14) willhave:

$$q'(x) + p(x)q(x) - r(x) \equiv 0$$

$$q''(x) + k(x)\mathrm{q}(\mathrm{x}) - \mathrm{p}(\mathrm{x}) \equiv 0 \qquad (17)$$

Theorem: Let one of the following conditions be satisfied in (1):

$$\text{A)}\ \ a_t \equiv 0,\ \ б)a_t \neq 0, \frac{b_t}{a_t} \neq 0, \text{b)} a_t \neq 0,\ \ \frac{b_t}{a_t} \equiv 0$$

Then for the existence of a solution of equations (1) independent
from the paramatra, it is necessary and sufficient that the coefficients, depending on the parameter, are connected by linear relationships: (4) for a); (9) for b); (12) at c); the coefficients of which are independent of the parameter and, in turn, are connected by differential relations:

(5) ata); (10) atб); (16) atв) and A≠ 0Where$\mathrm{q}(\mathrm{x}) = \mathrm{B}: \mathrm{A}$,

A, B is given by formulas (15), and the relation B≡0, for A≡0.
When relations (4), (5) or (9), (10), or (12), (16) are satisfied, equation (1) has a unique solution, and when relations B≡0, A≡0 are satisfied, the variety of solutions (1) contains one arbitrary constant.

LITERATURE

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