THE DEGENERATION OF THE SYSTEMS OF THE NONLINEAR EQUATIONS IN QUOTIENT DERIVED 2- ORDER WITH SINGULARS FACTOR

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ABSTRACT:

In this paper, we consider a system of three nonlinear equations with three singular coefficients. We study their compatibility conditions for the systems under study. Then there are some functions where they may be particular or special solutions of the system. Otherwise, we get that these systems will not be compatible. The goal of the work is to find the manifolds of solutions of system (1), the conditions, the compatibility of which are fulfilled identically. To find solutions to the problem, by making a replacement, we transform the original systems to the system of equations in full first-order differentials, find the solutions of the latter systems and the variety of their solutions. As a result of the integration of the systems, it was obtained that in what cases, and at what values, the n-solution of the original systems is continuous, and in other cases it is singular, moreover, the order of the singularity is shown.

Keywords: singular line - degeneration surfaces in three-dimensional spaces - variety of solutions - singular solutions-order singularities - removable singularity.

In the present paper, we consider an overdetermined system of second-order differential equations with singular coefficients that generalizes previous previously unstudied systems of the form:

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} = a_1(x,y,z)\cdot\dfrac{\partial u}{\partial x}, \\ \mathrm{x}^{\mathrm{n}}\cdot\dfrac{\partial^2 u}{\partial y\partial x} = a_2(x,y,z)\cdot\dfrac{\partial u}{\partial x} + b_1(x,y,z)\cdot\left(\dfrac{\partial u}{\partial x}\right)^k, \\ \mathrm{y}^{\mathrm{n}}\cdot\dfrac{\partial^2 u}{\partial z\partial x} = a_3(x,y,z)\cdot\dfrac{\partial u}{\partial x} + b_2(x,y,z)\cdot\left(\dfrac{\partial u}{\partial x}\right)^k, \end{cases} \quad (1)$$

where functions $a_j(x,y,z)$, и $b_r(x,y,z)$, $j=1,2,3$; r =1,2 . defined in the

parallelepiped

$$\overline{D} = \{0 \le |x|,|y|,|z| \le r_j\},\ j=(1,\ 2,\ 3), u\in C^3(D_0),$$

a, n are arbitrary positive and k-arbitrary real numbers. The system of equations (1) is equivalent to the following system of linear equations of the second order of the form

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} = a_1(x,y,z)\cdot\dfrac{\partial u}{\partial x}, \\ \dfrac{\partial^2 u}{\partial y \partial x} = \dfrac{a_2(x,y,z)}{x^n}\dfrac{\partial u}{\partial x} + \dfrac{b_1(x,y,z)}{x^n}\cdot\left(\dfrac{\partial u}{\partial x}\right)^k, \\ \dfrac{\partial^2 u}{\partial z \partial x} = \dfrac{a_2(x,y,z)}{yn}\cdot\dfrac{\partial u}{\partial x} + \dfrac{b_2(x,y,z)}{y^n}\cdot\left(\dfrac{\partial u}{\partial x}\right)^k, \end{cases} \tag{2}$$

After replacing, $\dfrac{\partial u}{\partial x} = V$ where V = V(x,y,z) is a new unknown function, we transform system (2) to a system of first-order equations, i.e. to the system of equations in full differentials by two unknown functions with three variables of the form.

$$\begin{cases} \dfrac{\partial V}{\partial x} = \dfrac{a_1(x,y,z)}{y^n}\cdot V, \\ \dfrac{\partial V}{\partial y} = \dfrac{a_2(x,y,z)}{x^n}\cdot V + \dfrac{b_1(x,y,z)}{x^n} V^k, \\ \dfrac{\partial V}{\partial z} = \dfrac{a_3(x,y,z)}{y^n}\cdot V + \dfrac{b_2(x,y,z)}{y^n}\cdot V^k. \end{cases} \tag{3}$$

The compatibility conditions of system (2) are as follows:

$$\begin{cases} \left(\dfrac{\partial}{\partial x}\left(\dfrac{a_2}{x^n}\right) - \dfrac{\partial a_1}{\partial y}\right)\cdot V + \left[\dfrac{\partial}{\partial x}\left(\dfrac{b_1}{x^n}\right) + (k-1)\dfrac{a_1 b_1}{x^n}\right]\cdot V^k = 0, \\ \left(\dfrac{\partial}{\partial x}\left(\dfrac{a_3}{y^n}\right) - \dfrac{\partial a_1}{\partial z}\right)\cdot V + \left[\dfrac{\partial}{\partial x}\left(\dfrac{b_2}{y^n}\right) + (k-1)\dfrac{a_1 b_2}{y^n}\right]\cdot V^k = 0 \\ \left(\dfrac{\partial}{\partial y}\left(\dfrac{a_3}{y^n}\right) - \dfrac{\partial}{\partial z}\left(\dfrac{a_2}{x^n}\right)\right)\cdot V + \left[\dfrac{\partial}{\partial y}\left(\dfrac{b_2}{y^n}\right) - \dfrac{\partial}{\partial z}\left(\dfrac{b_1}{x^n}\right) + (k-1)\dfrac{a_3 b_1 - a_2 b_2}{x^n y^n}\right]\cdot V^k = 0 \end{cases}$$

If condition (4) is mandatory, then the trivial and some other particular solutions of the system are obtained. Let these conditions be fulfilled identically. In the following way:

$$a_2(x,y,z) = x^n \cdot \left[\alpha_1(y,z) + A'_{1y}(x,y,z)\right],\ A_1(x,y,z) = \int_{x_0}^{x} a_1(t,y,z)dt \in C(\overline{D}),$$

$$a_3(x,y,z) = y^n \cdot \left[\alpha_2(y,z) + A'_{1z}(x,y,z)\right],\ b_1(x,y,z) = x^n \beta_1(y,z)\exp\{(1-k)A(x,y,z)\},$$

$$b_2(x,y,z) = y^n \beta_2(y,z)\exp\{(1-k)A(x,y,z)\}. \tag{5}$$

Moreover, system (1) is solvable, and the variety of its solutions is determined by the formula

$$u(x,y,z)=C_2(y,z)+\exp\{A(x,y,z)\}\cdot\int_0^x [C+B(t,y,z)]^{1/(1-k)}dt, \qquad (6)$$

where the integrand is expressed with respect to integral transformations of the coefficients of the original system, moreover, continuous in the entire region (including degeneration lines), and for it.

Theorem. *Suppose that in the system of equations (1) all functions and (), k = (1,2), are given functions in the box D. If the compatibility condition (4) is satisfied, but not identically, then system (1) has only a trivial solution. For the identical fulfillment of conditions (4), it is necessary and sufficient that the relationship between the coefficients of system (1) is determined by formulas of the form (5). Then system (1) is solvable and the variety of its solutions is determined by formula (6). Moreover, the obtained solution of the system in cases in the entire domain is continuous, and in cases in the domain D it has a logarithmic singularity and singularities of order (n-1) in cases n> 1.*

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