REGULARIZED ALGORITHMS FOR THE FORMATION OF CONTROL ACTIONS IN LOCALLY OPTIMAL CONTROL SYSTEMS FOR DYNAMIC OBJECTS

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ABSTRACT

Regularized algorithms for the formation of control actions in locally optimal control systems for dynamic objects are given. Given that the initial equations for estimating the parameters of the object and the control device are usually poorly conditioned, it becomes necessary to apply regular methods. Stable algorithms for finding the desired solutions are presented on the basis of regular nonorthogonal factorizations and pseudo-inversions of square matrices that contribute to an increase in the accuracy of the formation of control actions.

Keywords: closed control system, locally-optimal adaptive control system, control action, regularized algorithms.

INTRODUCTION

The main problem of parametric optimization consists in determining the parameters of the control algorithm from the condition of minimizing the chosen quality criterion [1-5]. Complex and time-consuming is the solution of the problem of parametric optimization for multimode objects or objects with slowly varying parameters when the rate of change of the parameters of the object is small and on the optimization interval they can be considered constant. The problem of parametric optimization can be reconciled with the general problem of synthesis of an adaptive control system [2,4,6]. In this case, both theoretical methods and numerical procedures are used to solve the problem of parametric optimization.

In the theoretical approach, adaptive control algorithms are defined, in which the parameters are functions of the coefficients of the mathematical model of the control object or depend on their specific relationships [4-7].

When using the second approach to the solution of the problem of parametric optimization, the control algorithm is known, and by modeling, the necessary ranges of the parameters of the control algorithm are determined, and on the basis of these results, functions for the adjustment of the parameters of the control algorithm are constructed. However, only a suboptimal solution can be obtained here [6-8].

Recursion algorithms [9-11] are most often used to estimate the coefficients of equations from observable data, allowing identification in the normal operation mode of the object. The control of the object leads to the degeneration of the information matrix and thereby prevents the identification of the object or the required optimal control law, determined by the identifiable parameters of the object.

There are various approaches to solving this problem. To prevent unidentifiability, various methods were proposed [7,10]: the addition of noise to the controller, the inclusion of several controllers in the control system and their connection by some algorithm, and others.In [7,12] the general form of control as a function of unknown parameters of a linear object is given, for which the problem of identifiability is removed in the sense that the nonidentifiability of the parameters of the object does not entail the unidentifiability of the required control. It is shown [7] that locally optimal control belongs to this species. In this connection, the identifiability of the optimal control law takes place in locally optimal control systems.

FORMULATION OF THE PROBLEM

Very often, when synthesizing a regulator in closed systems, methods of locally optimal adaptive control are used [5, 8, 12, 13]. Consider a linear control object described by equation

$$x_{t+1} = Ax_t + Bu_t + w_{t+1}, \tag{1}$$

where $x_t \in R^n$ – is the measured state, $\{w_t\}$ – is uncontrolled independent random perturbations satisfying condition

$$Ew_t = 0,\ Ew_t w_t^T = Q,$$

A and B are unknown matrices of dimension $n \times n$ and $n \times m$.

We take the control law in the form

$$u_t = \theta^T(\Omega_t)x_t,$$

where $\theta(\Omega)$ is the given matrix function, and Ω_t is the current estimate of the matrix $\Omega^T = [A \vdots B]$, obtained from the relations

$$\psi_{t+1} = \Omega^T \Phi_t + w_{t+1},$$

where Ω is an unknown matrix of dimension $n_\Phi \times n_\psi$; $\Phi_t \in R^{n_\Phi}$ and $\psi_t \in R^{n_\psi}$ are the vectors available to the measurement.

We define the sequence of estimations of matrices Ω according to the method of least squares on the basis of the recurrence relation [12]:

$$\Omega_{t+1} = \Omega_t + \Gamma_{t+1}^{-1}\Phi_t\left(\psi_{t+1} - \Omega_t^T\Phi_t\right)^T,$$

where Ω_0 is an arbitrary matrix of dimension $n_\Phi \times n_\psi$, and the matrix Γ_{t+1} has the form

$$\Gamma_{t+1} = \Gamma_t + \Phi_t\Phi_t^T,\ \Gamma_0 > 0,$$

$$\Phi_t^T = \left(x_t^T, u_t^T\right),\ \psi_{t+1} = x_{t+1}.$$

Then the limiting law of control will be determined by the expression [7,9]

$$\begin{gathered} u_t = \theta^T(A,B)x_t, \\ \theta^T(A,B) = (H^T B)^{-1} H^T (S^T - A). \end{gathered} \tag{2}$$

The control law (2) minimizes the conditional mathematical expectation of the value along the trajectory (1) of the objective function

$$V(x_{t+1}) = (x_{t+1} - x^{\theta}_{t+1})^T C (x_{t+1} - x^{\theta}_{t+1}),$$

while the nonnegative definite matrix $C = C^T$ satisfies the equality $H = CB$, and x^{θ}_{t+1} – is the state value of the reference trajectory determined by equation $x^{\theta}_{t+1} = S^T x_t$, and at $S = 0$ control (2) will coincide with locally optimal control.

Also consider the control object specified in the form

$$A(z^{-1}) y_{t+1} = B(z^{-1}) u_t + w_{t+1}, \tag{3}$$

where $y_t \in R^l$ - measured outputs, $u_t \in R^m$ - control.

We introduce the following notation

$$\Omega^T = (-A^{(n)}, \ldots, -A^{(1)}, B^{(n-1)}, \ldots, B^{(0)}),$$

$$\Phi^T_t = (y^T_{t-n+1}, \ldots, y^T_t,\ u^T_{t-n+1}, \ldots, u^T_t),$$

where $\psi_{t+1} = y_{t+1}$.

Then, taking into account that

$$u_t = \theta^T(\Omega_t)\eta_t,\quad \eta^T_t = (y^T_{t-n+1}, \ldots, y^T_t, u^T_{t-n+1}, \ldots, u^T_t),$$

the locally optimal control law takes the form:

$$\begin{gathered} u_t = (H^T B^{(0)})^{-1}[(A^{(1)} + S^{(1)}) y_t + \ldots \\ + (A^{(n)} + S^{(n)}) y_{t-n+1} + (-B^{(1)} + D^{(1)}) u_{t-1} + \ldots \\ + (-B^{(n-1)} + D^{(n-1)}) u_{t-n+1}], \end{gathered} \tag{4}$$

where $S^{(i)}$, $i = \overline{1, n}$ and $D^{(j)}$, $j = \overline{1, n-1}$ are arbitrary matrices of dimension $l \times l$ and $l \times m$ respectively, H – is the matrix $m \times l$ such that $\det H^T B^{(0)} \neq 0$.

III. SOLUTION OF THE TASK

Thus, for objects described by equations (1) and (3), a locally-optimal control law is formed on the basis of expressions (2) and (4), respectively. In the expressions (2) and (4), square matrices of the form $G = H^T B$ and $G^{(0)} = H^T B^{(0)}$ are inverted. The matrix data can be poorly conditioned, which ultimately leads to the necessity of constructing regular algorithms for the formation of the required estimates.In addition, in practical problems often elements, for example, of the matrix G are known to us approximately. In these cases, instead of the matrix G, we are dealing with some other matrix $\tilde{G}$ such that $\|\tilde{G} - G\| \le h$, where the meaning of the norms is usually determined by the nature of the problem. Having matrix G instead of matrix $\tilde{G}$, we can not, moreover, express a definite proposition on the degeneracy or nondegeneracy of the matrix G. Instead of such matrices, $\tilde{G}$ is infinitely large, and within the bounds of the known level of error they are indistinguishable. But there are infinitely many such matrices, and in the framework of the known error level they are indistinguishable. Among such "possible exact systems" there may be degenerate ones.

To give numerical stability to the procedure for inversion of matrices G and $G^{(0)}$, it is advisable to use the concepts of regular and stable estimation methods [14-16]. Below we present an algorithm for estimating the inverse matrix G^{-1} in equation (2). The same algorithm can also be used to estimate the matrix $G^{(0)^{-1}}$ in equation (4).

Suppose that the matrix $G = [g_{ij}],\ i, j = 1,2,\ldots,n$ is nondegenerate. Denote by G_k its left upper part, i.e. $G_k = [g_{ij}],\ i, j = 1,2,\ldots,k$. Matrices G_k, $k = 1,2,\ldots,n$, nondegenerate. Imagine them in the form

$$G^{(k)} = \left[\begin{array}{c|c} G_{11}^{(k-1)} & G_{12}^{(k)} \\ \hline G_{21}^{(k)} & G_{22}^{(k)} \end{array}\right],$$

$$k > 1,\ G_{11}^{(1)} = G_{22}^{(1)} = g_{11},\quad G_{12}^{(1)} = 0,\quad G_{21}^{(1)} = 0\,.$$

Following [17,18] of the method of bordering, the inverse matrix can be written in the following form

$$\left[G^{(k)}\right]^{-1} = \left[\begin{array}{c|c} P^{(k-1)} & R^{(k)} \\ \hline N^{(k)} & V^{(k)} \end{array}\right].$$

Then

$$V^{(k)} = \left[G_{22}^{(k)} - G_{21}^{(k)}\left[G_{11}^{(k-1)}\right]^{-1} G_{12}^{(k)}\right]^{-1},$$

$$\begin{aligned} R^{(k)} &= -V^{(k)}\left[G_{11}^{(k-1)}\right]^{-1} G_{12}^{(k)}, \\ N^{(k)} &= -V^{(k)} G_{21}^{(k)}\left[G_{11}^{(k-1)}\right]^{-1}, \end{aligned} \tag{5}$$

$$P^{(k-1)} = \left[G_{11}^{(k-1)}\right]^{-1} + V^{(k)}\left[G_{11}^{(k-1)}\right]^{-1} G_{12}^{(k)}\, G_{21}^{(k)}\left[G_{11}^{(k-1)}\right]^{-1}.$$

Carrying out the calculations using formulas (5) at $k = 2,3,\ldots,n$, one can obtain $G_n^{-1} = G^{-1}$.

Gauss method

In the case under consideration, it is advisable to use the Gauss method [17, 19-21] for matrix inversion, according to which a sequence of matrices $G^{(k)},\ k = 0,1,\ldots, G^{(0)} = G$ is constructed, in the form

$$G^{(k)} = \left[\begin{array}{c:c} G_{11}^{(k)} & G_{12}^{(k)} \\ \hdashline G_{21}^{(k)} & G_{22}^{(k)} \end{array}\right],$$

where $G_{11}^{(k)}$ is an invertible matrix of size $k \times k$.

To do this, the leading τs -nd element

$$\left|g_{\tau s}^{(k)}\right| = \max_{k<i\le n, k<j\le m} \left|g_{ij}^{(k)}\right|,$$

is searched for the existing cell $G_{22}^{(k)}$ at the *k*-th step, where $g_{ij}^{(k)}$ is the *ij*-th element of the matrix $G^{(k)}$.

If $\left|g_{\tau s}^{(k)}\right| > \varepsilon$, then the permutation of the τ -nd row and the *s*-rd column with the $(k+1)$-th row and the column of the matrix $G^{(k)}$ is done. Then the matrix $G^{(k+1)}$ is recalculated after the permutations

$$G_{22}^{(k)} = \left[\begin{array}{c:c} g_{k+1,k+1}^{(k)} & d^{(k)} \\ \hdashline \vartheta^{(k)} & W^{(k)} \end{array}\right],$$

$$G^{(k+1)} = \left[\begin{array}{c:c} G_{11}^{(k)} & G_{12}^{(k)} \\ \hdashline G_{21}^{(k)} & \left[\begin{array}{c:c} g_{k+1,k+1}^{(k)} & d^{(k)} \\ \hdashline \alpha^{(k)} & G_{22}^{(k+1)} \end{array}\right] \end{array}\right],$$

where $G_{22}^{(k+1)} = W^{(k)} - \alpha^{(k)} d^{(k)}$, $\alpha^{(k)} = \left(g_{k+1,k+1}^{(k)}\right)^{-1} \vartheta^{(k)}$.

If $\left|g_{\tau s}^{(k)}\right| \le \varepsilon$, then the factorization process stops and the non-orthogonal factorization of the matrix G in the form

$$G_\varepsilon = U_k R_k,\ U_k^T = \left[U_1^T \vdots U_2^T\right],$$

$$R_k = \left[R_1 \vdots R_2\right],\ U_2 = G_{21}^{(k)},$$

$$R_2 = G_{12}^{(k)},\quad U_1 = \begin{bmatrix} 1 & 0 & \dots & 0 \\ g_{21}^{(k)} & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ g_{k1}^{(k)} & g_{k2}^{(k)} & \dots & 1 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} g_{11}^{(k)} & g_{12}^{(k)} & \dots & g_{1k}^{(k)} \\ 0 & g_{22}^{(k)} & \dots & g_{2k}^{(k)} \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & g_{kk}^{(k)} \end{bmatrix}$$

is determined.

The approximation for the pseudoinverse matrix is thus constructed [17,21,22]:

$$G_{\varepsilon}^{+} = R_k^{+} U_k^{+} . \qquad (6)$$

The calculation of R^{+} and U^{+} in (6), when R and U^{T} are respectively upper-peptide matrices, is efficiently performed by orthogonal factorization $R = SP$ using the Givens or Householder transformations [18,20], where S – is the lower-triangular square, P – orthogonal matrix. Then $R^{+} = P^{T} S^{-1}$.

IV. Conclusion

The resulted regularized algorithms allow to stabilize the procedure of formation and development of control actions in locally optimal control systems for dynamic objects under conditions of poor conditionality of matrices G and $G^{(0)}$.

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