



ANISOTROPIC BACKGROUND WITH VARIABLE DECELERATION PARAMETER IN MODIFIED THEORY OF GRAVITY

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Abstract:

In the present investigation, LRS Bianchi type-I universe with variable deceleration parameter in the frame work of $f(R, T)$ theory of gravity is considered where $f(R, T)$ is an arbitrary function of Ricci scalar R and trace of the energy momentum tensor T . The gravitational field equations are obtained in the metric formalism, which follows from the covariant divergence of the stress-energy tensor. The field equations correspond for a specific choice of $f(R, T) = f_1(R) + f_2(T)$, with the individual superior functions $f_1(R) = \lambda_1 R$ and $f_2(T) = \lambda_2 T$. It is observe that in $f(R, T)$ gravity, an extra acceleration is always present due to coupling between matter and geometry. Allowing for time dependent deceleration parameter the solutions of the field equations and some physical and geometric properties of the model along with physical acceptability of the solutions have also been discussed in details.

Keywords: LRS Bianchi type-I universe, $f(R, T)$ theory of gravity, cosmological constant.

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I. Introduction:

The recent observations strongly indicate that our universe is spatially flat and has a phase transition from decelerating to accelerating [1-5]. It is well known that General Relativity (GR) based on the Einstein-Hilbert action. In order to explain the nature of dark energy and the accelerated expansion, various alternative theoretical models have been proposed such as quintessence, phantom energy, k-essence, tachyon, f-essence, chaplygin gas, etc. First alternative obtained by substituting Einstein-Hilbert term with an arbitrary function of the curvature scalar R , this is known as $f(R)$ theory of gravity. This theory has been widely studied by [6-7]. Harko et al. [8, 9] proposed a maximal extension of the Hilbert-Einstein action by assuming that the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and the matter Lagrangian L_m . It is also an extension of standard general relativity, so called $f(R, T)$ theory of gravity where T represents the trace of the stress-energy tensor. Within the framework of $f(R, T)$ gravity many authors [10-12] have investigated several aspects of this theory. Adhav [13] has obtained Bianchi type-I cosmological model in $f(R, T)$ gravity. Bianchi type-III and Kaluza-Klein cosmological models in $f(R, T)$ gravity has been discussed

by Reddy et al. [14, 15]. Katore et al. [16] investigated some cosmological model with dark energy source in $f(R, T)$ gravity. Chandell et al. [17], Chaubey et al. [18], Chirde et al. [19, 20] has studied some cosmological models in $f(R, T)$ gravity for different context of use.

The cosmological constant cold dark matter Λ CDM cosmological model is the simplest model of the universe that describes the present acceleration of universe and fits with the present day cosmological data [21]. The basic role of cosmological constant is related to the observational evidence of high red-shift Type Ia supernovae [1-5] for a small decreasing value of cosmological constant at the present epoch, without the cosmological constant there are no satisfactorily explanations like structure formation and the age of the universe in the cosmological model [22]. Several authors like [23-26] have discussed the cosmological scenarios with a time varying cosmological constant. Recently, Wankhade et al. [27] studied LRS Bianchi type-I Universe with anisotropic dark energy and special form of deceleration parameter in $f(R, T)$ Gravity and observed that the universe has a phase transition from decelerating to accelerating and remain present in Quintessence region. Motivated from the studies outlined above in the present paper, LRS Bianchi type-I cosmological model in the context of $f(R, T)$ gravity theory with cosmological term have been studied. The solutions of the Einstein's field equations have been obtained by applying variable deceleration parameter. This paper is organized as follows. Section II contains the brief review of $f(R, T)$ gravity formalism. Section III contains metric and the field equations. Solution of the field equations with the help of the deceleration parameter which is linear in time with a negative slope is given in section IV. Some physical and kinematical properties of the universe are examined in section V and VI. Section VII deals with brief discussion of presented model and finally some concluding remarks are drawn in sections VIII.

II. Gravitational Field Equations of $f(R, T)$ Gravity:

The action for modified theory of gravity of the form Harko et al. [9] is as

$$s = \frac{1}{16\pi} \int f(R, T) \sqrt{-g} d^4x + \int L_m \sqrt{-g} d^4x, \quad (1)$$

where $f(R, T)$ is an arbitrary function of the Ricci scalar R and of the trace T of the stress energy tensor of the matter.

The stress energy tensor of matter as Landau et al. [28] is

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_m)}{\delta g^{\mu\nu}}, \quad (2)$$

and its trace is given by $T = g^{\mu\nu} T_{\mu\nu}$.

Varying the action (1) with respect to the metric tensor components $g^{\mu\nu}$, the gravitational field equation of $f(R, T)$ gravity is obtained as

$$f_R(R, T) R_{ij} - \frac{1}{2} f(R, T) g_{ij} + f_R(R, T) (g_{ij} \nabla^i \nabla_j - \nabla_i \nabla_j) = 8\pi T_{ij} - f_T(R, T) T_{ij} - f_T(R, T) \Theta_{ij}, \quad (3)$$

where $f_R = \frac{\delta f(R, T)}{\delta R}$, $f_T = \frac{\delta f(R, T)}{\delta T}$ and $\Theta_{ij} = g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g^{ij}}$.

Here ∇_i is the covariant derivative and T_{ij} is usual matter energy momentum tensor derived from the Lagrangian L_m .

The contraction of equation (3) yields

$$f_R(R,T)R + 3\Pi f_R(R,T) - 2f(R,T) = (8\pi - f_T(R,T))T - f_T(R,T)\Theta \text{ with } \Theta = g^{\mu\nu}\Theta_{\mu\nu}. \quad (4)$$

Combining equation (3) and (4) and eliminating the $\Pi f_R(R,T)$ term gives

$$f_R(R,T)\left(R_{\mu\nu} - \frac{1}{3}Rg_{\mu\nu}\right) + \frac{1}{6}f(R,T)g_{\mu\nu} = (8\pi - f_T(R,T))\left(T_{\mu\nu} - \frac{1}{3}Tg_{\mu\nu}\right) - f_T(R,T)\left(\Theta_{\mu\nu} - \frac{1}{3}\Theta g_{\mu\nu}\right) + \nabla_\mu \nabla_\nu f_R(R,T), \quad (5)$$

where $\Theta_{\mu\nu} = -2T_{\mu\nu} - pg_{\mu\nu}$.

It is to mention here that this field equation depends on the physical nature of the matter field. Many theoretical models corresponding to different matter contributions for $f(R,T)$ gravity are possible. There are three classes of these models

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases}. \quad (6)$$

Bearing in mind all above classes Harko et al. [9] derived the gravitational field equations that may be relevant in explaining some of the open problems of cosmology and astrophysics. Also he had demonstrated the possibility of reconstruction of arbitrary FRW cosmologies by an appropriate choice of a function $f(T)$. Alvarenga et al. [29] studied the viability of $f(R,T)$ gravity giving to the energy conditions. He presented the general formalism of $f(R,T)$ theory by putting out $f(R,T) = f_1(R) + f_2(T)$, where $f_1(R)$ and $f_2(T)$ be the function of curvature and the trace of the energy momentum tensor. Also, by substituting suitable constraint on the input parameter he obtained $R + 2f(T)$ type model which satisfy the energy condition. Hence, in this paper I have focused on the second class, i.e. $f(R,T) = f_1(R) + f_2(T)$.

The gravitational field equation (3) becomes,

$$f_1'(R)R_{\mu\nu} - \frac{1}{2}f_1(R)g_{\mu\nu} + (g_{\mu\nu}\Pi - \nabla_\mu \nabla_\nu)f_1'(R) = 8\pi T_{\mu\nu} + f_2'(R)T_{\mu\nu} + \left(f_2'(T)p + \frac{1}{2}f_2(T)\right)g_{\mu\nu}, \quad (8)$$

where the prime denotes the differentiation with respect to the argument.

Here the particular form of the functions $f_1(R) = \lambda_1 R$ and $f_2(T) = \lambda_2 T$, where λ_1 and λ_2 are arbitrary constants is considered.

Without loss of generality, assume that $\lambda_1 = \lambda_2 = \lambda$ so that $f(R,T) = \lambda(R + T)$.

Thus equation (8) can be written as

$$\lambda R_{\mu\nu} - \frac{1}{2}\lambda(R,T)g_{\mu\nu} + (g_{\mu\nu}\Pi - \nabla_\mu \nabla_\nu)\lambda = 8\pi T_{\mu\nu} + \lambda T_{\mu\nu} + \lambda(2T_{\mu\nu} + pg_{\mu\nu}), \quad (10)$$

setting $(g_{\mu\nu}\Pi - \nabla_\mu \nabla_\nu)\lambda = 0$, one may obtain

$$\lambda G_{\mu\nu} = 8\pi T_{\mu\nu} + \lambda T_{\mu\nu} + \left(\lambda p + \frac{1}{2} \lambda T \right) g_{\mu\nu}, \quad (11)$$

Which on rearranging gives

$$G_{\mu\nu} - \left(p + \frac{1}{2} T \right) g_{\mu\nu} = \left(\frac{8\pi + \lambda}{\lambda} \right) T_{\mu\nu}. \quad (12)$$

We have the Einstein field equation with cosmological constant as

$$G_{\mu\nu} - \Lambda g_{\mu\nu} = -8\pi T_{\mu\nu}. \quad (13)$$

By choosing a negative small value for the arbitrary λ so as to have the same sign of the RHS of equation (12)

and (13), I have kept this choice of λ throughout. The term $\left(p + \frac{1}{2} T \right)$ can now be regarded as a cosmological constant. So in the framework of the $f(R, T)$ gravity, one can get the cosmological constant as a function of the equation of state parameter ω , the energy density ρ and the trace of the stress-energy tensor T . Since ω and ρ are already included in T so one could just write

$$\Lambda = \Lambda(T) = p + \frac{1}{2} T. \quad (14)$$

The dependence of the cosmological constant Λ on the trace of the energy momentum tensor T has been proposed by Poplawski [30] where the cosmological constant in the gravitational Lagrangian is a function of the trace of the energy-momentum tensor and considering the perfect fluid the trace of this presented model is $T = -3p + \rho$.

III. Metric and Field Equations:

A Bianchi Type-I cosmological model, being the generalization of flat Friedmann-Robertson-Walker (FRW) model which is one of the simplest models of the anisotropic universe. Therefore I confine myself to Bianchi type-I model in the context of $f(R, T)$ gravity.

$$ds^2 = dt^2 - A^2 dx^2 - B^2 (dy^2 + dz^2), \quad (15)$$

where the metric potentials A and B be the functions of time t only.

In case of a radial symmetry between the metric potentials, universe (15) is equal to FRW universe.

Some parameters for the LRS Bianchi type-I universe which are important in cosmological observations are given as follows.

The average scale factor and spatial volume respectively are defined as

$$a = \left(AB^2 \right)^{1/3}, \quad V = a^3 = \left(AB^2 \right), \quad (16)$$

The anisotropy parameter of the expansion is expressed as

$$A_m = \frac{1}{3} \sum_{i=1}^3 \left(\frac{\Delta H_i}{H} \right)^2, \quad (17)$$

where

$$H = (\ln a)_4 = \frac{a_4}{a} = \frac{1}{3} \left(\frac{A_4}{A} + 2 \frac{B_4}{B} \right), \tag{18}$$

be the mean Hubble parameter and $H_i (i=1, 2, 3)$ represent the directional Hubble parameters in the directions of x, y and z axes respectively.

The kinematical parameters which are observational interest in cosmology to define the behavior of the universe are the expansion scalar θ , the shear scalar σ^2 and the deceleration parameter q which are respectively given as

$$\theta = u^m_{;m} = \left(\frac{A_4}{A} + 2 \frac{B_4}{B} \right), \tag{19}$$

$$\sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} = \frac{1}{3} \left(\frac{A_4}{A} - \frac{B_4}{B} \right)^2, \tag{20}$$

$$q = -\frac{a_{44} a}{a_4^2} = \frac{d}{dt} \left(\frac{1}{H} \right) - 1. \tag{21}$$

The equation of motion (13) for the LRS Bianchi type-I universe (15) with the fluid of stress energy tensor can be written as

$$2 \frac{B_{44}}{B} + \frac{B_4^2}{B^2} = \left(\frac{8\pi + \lambda}{\lambda} \right) \rho - \Lambda, \tag{22}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = \left(\frac{8\pi + \lambda}{\lambda} \right) \rho - \Lambda, \tag{23}$$

$$2 \frac{A_4 B_4}{AB} + \frac{B_4^2}{B^2} = \left(\frac{8\pi + \lambda}{\lambda} \right) (-\rho) - \Lambda, \tag{24}$$

where the suffice (4) denotes the derivative with respect to time t .

IV. Solution of Field Equations:

Equations (22) to (24) are non linear system of three independent equations with five unknowns. In order to solve the system completely one more extra condition is needed. To do this assume the condition proposed by

Collins et al. [31] i.e. $\frac{\sigma}{H}$ is constant which gives the relation between the metric potentials as

$$A = B^n. \tag{25}$$

According to the work of Akarsu and Dereli [32], the deceleration parameter which is linear in time with a negative slope is considered and is given by equation (21).

As time dependence of the scale factor reflects main events in history of the Universe. Moreover it is the deceleration parameter who dictates the expansion rate of the Hubble sphere and determines the dynamics of the observable galaxy number variation: depending on the sign of the deceleration parameter this number either grows (in the case of decelerated expansion), or we are going to stay absolutely alone in the cosmos (if the expansion is accelerated).

One can classify models of Universe on the basis of time dependence of the two parameters. All models can be characterized by whether they expand or contract, and accelerate or decelerate:

- (a) $H > 0, q > 0$: expanding and decelerating
- (b) $H > 0, q < 0$: expanding and accelerating

- (c) $H < 0, q > 0$: contracting and decelerating
- (d) $H < 0, q < 0$: contracting and accelerating
- (e) $H > 0, q = 0$: expanding, zero deceleration
- (f) $H < 0, q = 0$: contracting, zero deceleration
- (g) $H = 0, q = 0$: static

Integrating equation (21) gives the average scale factor as

$$a(t) = \exp \int \frac{dt}{[(1+q)dt + \gamma]} \tag{26}$$

Where γ be the arbitrary constant. It is an easy choice that provides $a(t)$ as an explicit function of time for constant value of deceleration parameter. But, when q is taken to vary with time, an explicit determination of $a(t)$ leads to a possible choice of q as Abdussattar and Prajapati [33]

$$q = -\frac{\alpha}{t^2} + (\beta - 1) \tag{27}$$

Here $\alpha > 0$ is a parameter having the dimension of square of time and $\beta > 1$ is dimensionless constant. Obviously, the different values of α and β will give rise to different models. For $\beta > 1$ the model shows decelerating behavior but for $\beta \leq 1$ it shows accelerating behavior. Equation (26) can be integrated to give the time variation of the scale factor as

$$a(t) = \exp \left[\frac{1}{\beta} \int \frac{dt}{\left(t^2 + \frac{\gamma}{\beta} + \frac{\alpha}{\beta} \right)} \right] \tag{28}$$

Setting $\gamma = 0$ and integrating, the average scale factor $a(t)$ is obtain as

$$a(t) = \left(t^2 + \frac{\alpha}{\beta} \right)^{\frac{1}{2\beta}} \tag{29}$$

Using equations (16), (25) and (29) the metric potential can be obtained as

$$A = \left(t^2 + \frac{\alpha}{\beta} \right)^{\frac{3n}{2\beta(n+2)}} \tag{30}$$

$$B = \left(t^2 + \frac{\alpha}{\beta} \right)^{\frac{3}{2\beta(n+2)}} \tag{31}$$

Equations (30) and (31) gives

$$ds^2 = dt^2 - \left(t^2 + \frac{\alpha}{\beta} \right)^{\frac{3n}{\beta(n+2)}} dx^2 - \left(t^2 + \frac{\alpha}{\beta} \right)^{\frac{3}{\beta(n+2)}} (dy^2 + dz^2) \tag{32}$$

It is observed that at the initial time $t=0$ that is when the universe starts to expand, both the directional scale factors A and B remains constant and for $n=1$ this derived model approaches to isotropy universe.

V. Physical Properties of the Universe:

Physical parameters are important to describe the physical behavior of the universe. Solving equations (22) to (24) and using equations (30) and (31), the anisotropic pressure p , energy density ρ and cosmological constant Λ respectively are obtained as

$$p = \frac{\lambda}{(8\pi + \lambda)} \left\{ \left(\frac{6}{\beta(n+2)} - \frac{\lambda}{2\beta(4\pi + \lambda)} \right) \left(t^2 + \frac{\alpha}{\beta} \right)^{-1} + \left[\frac{18 - 24\beta(n+2)}{2\beta^2(n+2)^2} - \frac{\lambda[(3-2\beta)n^2 + (18-8\beta)n - 8\beta + 6]}{2\beta^2(n+2)^2(4\pi + \lambda)} \right] t^2 \left(t^2 + \frac{\alpha}{\beta} \right)^{-2} \right\}, \quad (33)$$

$$\rho = \frac{\lambda^2}{2\beta(4\pi + \lambda)(8\pi + \lambda)} \left\{ \left(t^2 + \frac{\alpha}{\beta} \right)^{-1} + \left[\frac{[(3-2\beta)n^2 - (n+1)8\beta - 3]}{\beta(n+2)^2} \right] t^2 \left(t^2 + \frac{\alpha}{\beta} \right)^{-2} \right\}, \quad (34)$$

$$\Lambda = \frac{-\lambda}{2\beta(4\pi + \lambda)} \left\{ \left(t^2 + \frac{\alpha}{\beta} \right)^{-1} + \left[\frac{(3-2\beta)n^2 + (18-8\beta)n + 6}{\beta(n+2)^2} \right] t^2 \left(t^2 + \frac{\alpha}{\beta} \right)^{-2} \right\}. \quad (35)$$

VI. Kinematical Properties of the Universe:

The kinematical properties which are important in cosmology for discussing the geometrical behavior of the universe that are spatial volume V , Hubble parameter H , expansion scalar θ , shear scalar σ^2 , and anisotropic parameter A_m are respectively obtained as follows.

$$V = \left(t^2 + \frac{\alpha}{\beta} \right)^{\frac{3}{2\beta}}. \quad (36)$$

It is observed that initially at $t=0$ the spatial volume V remains constant and expands exponentially as t increases and becomes infinitely large at $t = \infty$.

$$H = \frac{t}{\beta} \left(t^2 + \frac{\alpha}{\beta} \right)^{-1}. \quad (37)$$

$$\theta = \frac{3t}{\beta} \left(t^2 + \frac{\alpha}{\beta} \right)^{-1}. \quad (38)$$

$$\sigma^2 = \frac{(n-1)t}{\beta(n+2)} \left(t^2 + \frac{\alpha}{\beta} \right)^{-1}. \quad (39)$$

$$A_m = \frac{3(n-1)^2}{(n+2)^2}. \quad (40)$$

Deceleration parameter for this derived model is obtained as

$$q = (\beta - 1) - \frac{\alpha}{t^2}. \quad (41)$$

VII. Discussion:

The asymptotic behavior of the parameters of presented model is as follows.

Parameters	$t \rightarrow 0$	$t \rightarrow \infty$
A, B, a	Constant	∞
ρ	∞	0
Λ	∞	0
V	Constant	∞
H	∞	0
σ^2	∞	0
A_m	Constant	
q	∞	$\beta - 1$

- From the table, it is observed that the spatial volume V and the directional scale factors A and B are remains constant as $t \rightarrow 0$. However the volume scale factor expands exponentially as t increases and becomes infinitely large at $t = \infty$ which shows the late time acceleration of the universe.
- The other parameters such as energy density ρ and the mean Hubble parameter H diverges as $t \rightarrow 0$. Thus the model exhibits initial singularity at $t = 0$ [34].
- The model has high shear in the beginning of the evolution of the universe and for large time it tends to zero. Also it is observed that the mean anisotropy parameter is not dependent on the cosmic time t and it remains constant throughout the evolution of the universe from initial to infinite expansion.
- It is found that for $n = 1$, $\sigma^2 = 0$ and $A_m = 0$ which implies that our universe is homogeneous, isotropic and shear free. It is interesting to note that for $n = 1$ the physical behavior of shear scalar and mean anisotropic parameter resembles with the results obtained by Reddy et al. [35].
- From figure (i) it is observed that at the initial stage from where the model starts to expand, for small interval of time i.e. for $0.1 \leq t \leq 2.1$, the energy density increases slightly whereas with the expansion for the interval $2.2 \leq t \leq 12.9$ it decreases rapidly and at infinite time $t \rightarrow \infty$ the energy density approaches to zero i.e. $\rho \rightarrow 0$. Thus the model is asymptotically empty.
- The sign of deceleration parameter q indicates whether the universe inflates or not. In this derived model, the inflation of universe is depend on the value of β . For $\beta > 1$, the mode of universe is accelerating which is consistent with the recent CMB and WMAP observations

whereas for $\beta < 1$ it is decelerating which is also consistent with the high red-shift supernovae-Ia data [1-5]. Again for $\beta = 1$ the mode of universe is constant exponent (de-Sitter expansion). It is interesting to note that this result is resemblance with the work of Wankhade et al. [27].

- Figure (ii) and (iii) be the plot of cosmological term Λ versus time t for $\lambda = 1$ and $\lambda = -1$ respectively. Recent cosmological observations [1-5, 36-38] suggest the existence of a positive cosmological constant Λ . The observations on red-shift of type Ia supernova suggests that our universe may be an accelerating one with induced cosmological density through the cosmological Λ term.
- From figure (ii), it is observed that initially for small interval of time $0.1 \leq t \leq 2.9$ the cosmological term Λ is increases with expansion while for whole expansion it decreases positively and approaches to a small positive value (i.e. at present epoch) which corresponds to a negative effective mass density (repulsion) which shows the expansion will tends to accelerate. Thus, the nature of Λ in this derived model for $\lambda = -1$ is supported by recent observations. For $\lambda = 1$ a cosmological term attains a negative value for the time interval $0.3 \leq t \leq 3.3$, thus this derived model becomes vacuum model which is a simplest candidate of dark energy. Also for $t \geq 3.4$ it attains the value which is greater than -1 which represent a quintessence Λ CDM model. For this value the expansion will slow down, stop and reverse.

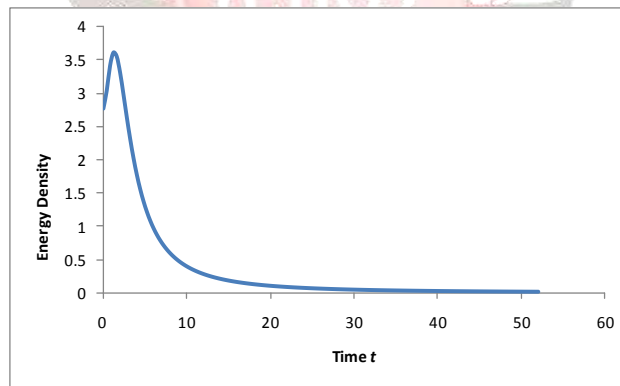


Fig.(i): Energy density verses time t for $\lambda = 1$

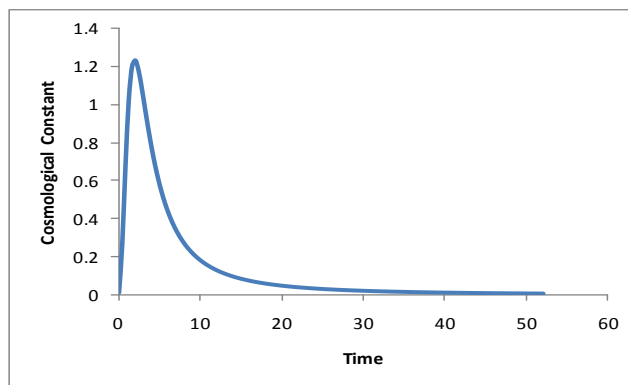


Fig.(ii): Cosmological constant verses time t for $\lambda = -1$

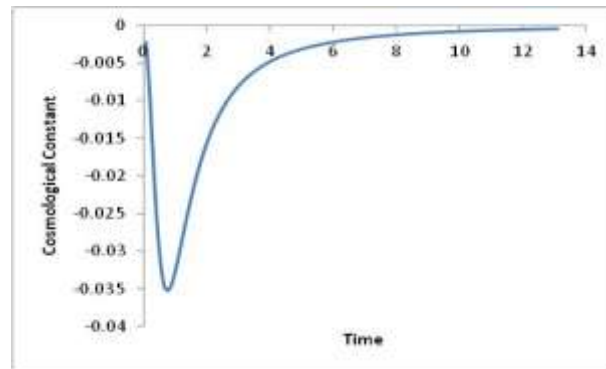


Fig.(iii): Cosmological constant verses time t for $\lambda=1$

VIII. Conclusions:-

In this paper a spatially homogeneous and anisotropic LRS Bianchi type-I cosmological model with perfect fluid in the context of $f(R,T)$ theory of gravity with cosmological constant Λ is discussed. The solution of the field equations is obtained towards variable deceleration parameter which generates a transition of the universe from the early decelerating phase to the recent accelerating phase. It has been found that for $\lambda=-1$ a cosmological constant is a decreasing function of time and it converges to a small positive value at late time which is responsible for the accelerated expansion of the universe which is supported by recent cosmological observations whereas for $\lambda=1$ it is negative for which the expansion will slow down, stop and reverse. The considered average scale factor leads deceleration parameter to be variable is physically justified with the recent observations. The model represents an expanding, shearing, non-rotating and accelerating universe. For $n \neq 1$ the present universe maintain its anisotropy but for suitable choice of constants $n=1$ the anisotropic parameter A_{mn} tends to zero. Hence, for this particular value of n the present model becomes isotropic. It is interesting to note that in $f(R,T)$ gravity, an extra acceleration is always present due to coupling between matter and geometry.

Conflict of Interests: The authors declare that there is no conflict of interests regarding the publication of this paper.

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