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ABOUT THE PROPERTY OF CONTROLLABILITY AN ENSAMBLE OF TRAJECTORIES OF DIFFERENTIAL INCLUSION

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ABSTRACT

In this paper the property of controllability an ensemble of trajectories of differential inclusion with control parameter is researched. The control problem of an ensemble trajectories from the initial state x^0 to a given terminal set $Y = Y(t)$ are studied. The necessary and sufficient conditions for "point" (x^0, Y)-controllability and completely Y-controllability are given.

Keywords: differential inclusion, control system, ensemble of trajectories, controllability conditions.

INTRODUCTION

The modern theory of differential inclusions and their applications are developing in several directions. Differential inclusions have important applications in the mathematical theory of optimal control, in the theory of differential equations with discontinuous right-hand sides, in differential games, in models of economic dynamics, and in other fields [3,5,11,12,15,16].

The control and observation under the conditions of informational limitations (incomplete data) arise as a result of taking into account such important factors as measurement errors, incomplete and delayed information about the initial data and external disturbances, etc. These problems are studied using mathematical models of control systems under conditions of uncertainty [2, 3, 4]. In studies of such models, differential inclusions with control parameters (controlled differential inclusions) and their discrete analogues are used as an effective mathematical apparatus [6,7,8,13,14].

For control systems under conditions of uncertainty, the properties of the ensemble of trajectories, methods for estimating the reachability set and forecasting the phase state of the system and others are of great interest [2,3,4]. Depending on the criterion for assessing the state of the system, various problems of optimal control of the ensemble of trajectories are studied: control by speed, minimax, and other criteria [8,9,17].

The problem of controllability, the essence of which is about the possibility of transferring the system from a given initial state to the desired terminal (final) state, is important for each model of a dynamic control system. Here, of great theoretical interest are the necessary and sufficient conditions for controllability. Such conditions have been studied for individual classes of control systems. Separate methods for constructing controls solving this problem have been developed. More complete results were obtained for the determine models of control systems. The study of each control problem for differential inclusions is based on the undamental properties of such dynamical systems. The properties of controlled differential inclusions and some optimization problems for such systems were studied in [6,8,9].

The control problem of ensemble of trajectories of differential inclusions can have various statements. The control problem for such systems can be posed, for example, as the problem of the complete or partial immersion of the end points of all possible trajectories of a system on a given terminal set. Some statements of controllability problems for differential inclusions, understood as controllability of an ensemble of trajectories with respect to given initial and terminal states, were considered in [9, 10,17,18]. For such systems, the controllability conditions and some properties of the set of controllability points with respect to the terminal set are studied.

STATEMENT OF THE PROBLEM

Consider a controlled differential inclusion [9, 17]

$$\dot{x} \in F(t,x,u), t \geq t_0, \qquad (1)$$

where $\dot{x} = \dfrac{dx}{dt}$, $x \in R^n$, $F(t,x,u) \subset R^n$, $u \in R^m$. Here, the parameter u plays the role of control actions. By an admissible control for system (1) we mean a measurable bounded m - vector function $u = u(t)$ defined on a certain interval $T = [t_0, t_1]$ of time. Denote $U_T(L)$ - the set of all admissible controls $u(t), t \in T$, with values from a closed ball $S_L = \{v \in R^m : \|v\| \leq L\}$. We denote by $H_T(u, x^0)$ the set of absolutely continuous solutions $x = x(t)$, $t \in T$, differential inclusion (1) corresponding to the control $u \in U_T(L)$ and the initial condition $x(t_0) = x^0$, where $x^0 \in R^n$.

Consider the set $X_T(t,u,x^0) = \{\xi \in R^n : \xi = x(t), x(\cdot) \in H_T(u,x^0)\}$. The multi-valued mapping $t \to X_T(t,u,x^0)$ is called the ensemble of trajectories of the system (1).

We consider the controllability problem for differential inclusion (1) in the sense of complete immersion of the ensemble of trajectories on a given a convex closed terminal set $Y = Y(t) \subset R^n$, $t \geq t_0$.

Definition 1. *We say that the ensemble of trajectories of system (1) is controllable from the initial state* $x^0 \notin Y(t_0)$ *to the set of terminal states* $Y = Y(t)$ *(system (1) is* (x^0, Y) *- controllable) if there exist is an admissible control* $u = u(t)$, $t \in T = [t_0, t_1]$ *such that the corresponding ensemble of trajectories satisfies the boundary condition*

$$X_T(t_1, u, x^0) \subset Y(t_1). \qquad (2)$$

Definition 2. *An ensemble of trajectories of system (1) is called completely controllable into a terminal set* $Y = Y(t)$ *(system (1) is completely Y- controllable) if the system is* (x^0, Y) *- controllable for each initial points* $x^0 \notin Y(t_0)$.

From Definition 1 it is clear that (x^0, Y) – the controllability of the system (1) means the solvability of the boundary value problem (1) - (2) in the class of admissible controls $u = u(t)$, $t \in T = [t_0, t_1]$. Clarification of the conditions for the solvability of this problem is the main goal of the study provided for in this paper.

THE MAIN RESULTS

It is clear from the above definitions that for the controllability of the ensemble of trajectories of system (1), the conditions for compactness and convexity of the set $X_T(t, u, x^0)$, as well as the continuous dependence of $X_T(t, u, x^0)$ on (t, u) are essential. Therefore, with respect to the right-hand side $F(t, x, u)$ of differential inclusion (1), we impose some conditions.

Assumption 1.

1) for any $(t, x, y, u) \in T_\infty \times R^n \times R^m$ the set $F(t, x, u)$ convex compact from R^n ($T_\infty = [t_0, \infty]$);

2) the multi-valued map $(t, x, u) \to F(t, x, u)$ is measurable in $t \in T_\infty$ for $\forall (x, u) \in R^n \times R^m$ and continuous in (x, u) for almost all $t \in T_\infty$;

3) the multi-valued mapping $x \to F(t, x, u)$ satisfies the Lipschitz condition: $h(F(t, x', u), F(t, x'', u)) \le l(t, u)\|x' - x''\|$, $\forall x', x'' \in R^n$, where the function $l(t, u)$ is such that $l(t, u(t))$ is summable on $T = [t_0, t_1]$ for any admissible control $u = u(t)$, $t \in T = [t_0, t_1]$($h(F_1, F_2)$– Hausdorf metric);

4) there are functions $g_i(t, u)$, $i = 1,2,$ such that $g_i(t, u(t))$, $i = 1,2,$ summable on T functions for any admissible controls $u = u(t)$, $t \in T = [t_0, t_1]$, $T \subset T_\infty$, and it is true $\|\xi\| \le g_1(t, u)\|x\| + g_2(t, u), \forall \xi \in F(t, x, u)$, $(t, x, u) \in T_\infty \times R^n \times R^m$.

5) the support function $C(F(t, x, u), \psi) = \max\{(\xi, \psi) : \xi \in F(t, x, u)\}$ is concave in x for almost all $t \in T_\infty$ and all $(u, \psi) \in R^m \times R^n$.

From the results of [17] it easily follows

Lemma 1. *Let Assumption 1 hold. Then:*

A) for any $u \in U_T(L)$, $x^0 \in R^n$, *and* $t \in T = [t_0, t_1]$, *the set* $X_T(t, u, x^0)$ *is a non-empty convex compact set from* R^n;

B) the multi-valued map $(t, u) \to X_T(t, u, x^0)$ *is continuous on* $T \times U_T(L)$ *in the metric* $R^1 \times L_2(T)$, *where* $L_2(T)$ *is the space* of square summable functions.

Let the right-hand side of differential inclusion (1) have the form

$$F(t, x, y, u) = A(t)x + b(t, u), \tag{4}$$

i.e. consider the following differential inclusion:

$$\dot{x} \in A(t)x(t) + b(t, u(t)), \tag{5}$$

where $A(t)$ is square matrix of size *n*, $b(t, u)$ is a nonempty subset of R^n. The following conditions will be imposed on the right-hand side of differential inclusion (5):

Assumption 2.

1) elements of $n \times n$ -matrix $A(t)$ are summable on any $T = [t_0, t_1] \subset T_\infty$;

2) for any $(t, u) \in T_\infty \times R^m$, the set $b(t, u)$ is a compact from R^n;

3) the multi-valued mapping $(t, u) \to b(t, u)$ is measurable in $t \in T_\infty$ and continuous in $u \in R^m$, moreover, $\|\xi\| \le \beta_1(t)\|u\| + \beta_2(t)$, $\forall \xi \in b(t, u)$, $(t, u) \in T_\infty \times R^m$, where $\beta_i(\cdot)$, $i = 1,2,$, are functions summable on any interval $T \subset T_\infty$.

4) the support function $C(b(t, u), \psi) = \max\{(\xi, \psi) : \xi \in b(t, u)\}$ is convex in $u \in V$ for almost all $t \in T_\infty$.

Under the conditions of Assumption 2, the following representation of the ensemble of trajectories of system (5) is valid through its parameters [17]:

$$X_T(t, u, \varphi^0) = F(t, t_0)x^0 + \int_{t_0}^{t} F(t, \tau) b(\tau, u(\tau)) d\tau, \tag{6}$$

where $F(t, \tau) - n \times n$ is a matrix function satisfying the equation

$$\frac{\partial F(t,\tau)}{\partial \tau} = -F(t,\tau)A(\tau),\ \tau \le t,\ F(t,t) = E,$$

E– identity $n \times n$ - matrix.

If the conditions of Assumption 2 are satisfied, then the multivalued mapping $(t,x,u) \to F(t,x,u)$ of the form (4) satisfies the conditions of Assumption 1. Therefore, for the control system (5), all the statements of Lemma 1 remain valid. Using the representation of the ensemble of trajectories of the linear system (5), and taking into account the properties of the integral of multivalued mappings, we verify that the convexity property of each set $X_T(t,u,x^0)$, $t \in T = [t_0, t_1]$ is preserved without requiring the convexity of the values of the multivalued mapping $(t,u) \to b(t,u)$.

Further, using formula (6) and the properties of the support functions, we have:

$$C(X_T(t,u,x^0),\psi) = (F(t,t_0)\varphi^0(t_0)) + \int_{t_0}^{t} C(F(t,\tau)b(\tau,u(\tau)),\psi)d\tau \ . \qquad (7)$$

From this formula for the support function of the set $X_T(t,u,\varphi^0)$ it easily follows that the support function $C(X_T(t,u,\varphi^0),\psi)$ is convex in $u \in U_V(T)$ for all $t \in T$ and all $\psi \in R^n$.

Therefore, the following statement is true.

Lemma 2. *Let Assumption 2 be satisfied. Then all the statements of Lemma 1 are true and, moreover, the support function*

$$C(X_T(t,u,\varphi^0),\psi) = \max\{(\xi,\psi) : \xi \in X_T(t,u,\varphi^0)\}$$

is convex in $u \in U_T(L)$ *for all* $t \in T$ *and all* $\psi \in R^n$.

According to Definition 1, system (1) (x^0, Y) -controllable if and only if inclusion system: $X_T(t_1,u,\varphi^0) \subset Y(t_1),\ u \in U_T(L)$ is compatible. Therefore, by virtue of Lemma 2 and the results of [9], the following statement is true.

Theorem 1. *For* (x^0, Y) *- controllability of system (5) it is necessary and sufficient that there exists* $t_1 > t_0$ *such that*

$$\sup_{\|\psi\|=1}\{(F(t_1,t_0)x^0,\psi) + \inf_{u \in U_T(L)} \overline{conc}_{\psi}\left[\int_{t_0}^{t_1} C(F(t_1,t)b(t,u(t)),\psi) - C(Y(t_1),\psi)\right] \le 0 \qquad (8)$$

where $\overline{conc} f$ is the concave closure [2] of the function *f*.

The above theorem gives a criterion for controllability of the ensemble of trajectories in the form of relation (8). Theorem 1 will be the theoretical basis for the subsequent results we obtained on controllability conditions.

Let in (5) $b(t,u) = B(t)u + Q(t)$, where $B(t)$ is a $n \times m$ -matrix, $Q(t)$ is a nonempty subset of R^n, i.e. we consider a linear controlled differential inclusion

$$\dot{x} \in A(t)x(t) + B(t)u(t) + Q(t) \qquad (9)$$

Assumption 3.

1) elements of $n \times m$ -matrix $B(t)$ are summable on any $T = [t_0, t_1] \subset T_\infty$;

2) $Q(t)$ – convex closed and bounded subsets R^n;

3) the multi-valued mapping $t \to Q(t)$, $t \geq t_0$ is measurable.

Under Assumption 3, all conditions of Assumption 2 are satisfied. Therefore, since

$$\inf_{u \in U_T(L)} \overline{conc}_{\psi} \left[\int_{t_0}^{t_1} C(F(t_1,t)b(t,u(t)),\psi) - C(Y(t_1),\psi) \right] =$$

$$= -L\int_{t_0}^{t_1} \|B'F'(t_1,t)\psi\| dt + \overline{conc}_{\psi} \left[\int_{t_0}^{t_1} C(F(t_1,t)Q(t),\psi)dt - C(Y(t_1),\psi) \right],$$

then from Theorem 2 it follows that controllability criterion for the ensemble of trajectories of system (9) have the form:

$$\sup_{\|\psi\|=1}\{(F(t_1,t_0)x^0,\psi) - L\int_{t_0}^{t_1} \|B'F'(t_1,t)\psi\| dt + \overline{conc}_{\psi} \left[\int_{t_0}^{t_1} C(F(t_1,t)Q(t),\psi)dt - C(Y(t_1),\psi) \right] \leq 0.$$

(10)

We introduce the notation:

$$P(t) = \int_{t_0}^{t} F(t,\tau)Q(\tau)d\tau,\ t > t_0.\ \ p^0(t) = F(t,t_0)x^0.$$

Then we have: $(F(t_1,t_0)x^0,\psi) + \overline{conc}_{\psi} \left[\int_{t_0}^{t_1} C(F(t_1,t)Q(t),\psi)dt - C(Y(t_1),\psi) \right] =$

$$= (p^0(t_1),\psi) - \overline{co}_{\psi}[C(Y(t_1),\psi) - C(P(t_1),\psi)] = (p^0(t_1),\psi) - C(Y(t_1) \underline{*}\, P(t_1),\psi),$$

where $Y(t_1) \overset{*}{-} P(t_1) = \{\xi \in R^n : \xi + P(t_1) \subset Y(t_1)\}$ is the geometric difference of the sets $Y(t_1)$ and $P(t_1)$. Therefore, relation (10) takes the form:

$$\inf_{\|\psi\|=1} \{L\int_{t_0}^{t_1} \|B'F'(t_1,t)\psi\| dt + C(Y(t_1) \overset{*}{-} P(t_1),\psi) - (p^0(t_1),\psi)\} \geq 0. \tag{11}$$

Thus, we have obtained the following controllability criterion for the ensemble of trajectories of system (9) into the terminal set $Y = Y(t)$.

Theorem 2. *The ensemble of trajectories of system (9) is (x^0, Y) - controllable if and only if relation (11) holds, where $t_1 > t_0$ and $L > 0$.*

Now, using this result, we will find out the conditions for the complete Y – controllability of system (9).

Theorem 3. *Let there exist $t_1 > t_0$ such that $Y(t_1) \overset{*}{-} P(t_1) \neq \varnothing$ and*

$$\lambda \equiv \inf_{\|\psi\|=1} \int_{t_0}^{t_1} \|B'(t)F'(t_1,t)\psi\| dt > 0. \tag{12}$$

Then system (9) is completely Y – controllable.

Proof. According to Theorem 2, it is enough for us to show that relations (11) holds for each initial points $x^0 \notin Y(t_0)$, and for some $t_1 > t_0$. We have:

$$\inf_{\|\psi\|=1} \{L\int_{t_0}^{t_1} \|B'F'(t_1,t)\psi\| dt + C(Y(t_1) \overset{*}{-} P(t_1),\psi) - (p^0,\psi)\} \geq$$

$$\geq L \inf_{\|\psi\|=1} \int_{t_0}^{t_1} \|B'F'(t_1,t)\psi\| dt + \inf_{\|\psi\|=1} C(Y(t_1) \overset{*}{-} P(t_1),\psi) + \inf_{\|\psi\|=1} (p^0,\psi) \geq$$

$$= L\lambda + \inf_{\|\psi\|=1} C(Y(t_1) \overset{*}{-} P(t_1),\psi) - \|p^0\|. \tag{13}$$

Since, by condition (12) $\lambda > 0$, then for

$$L \geq \max\left\{0, \frac{\|p^0\| - \inf_{\|\psi\|=1} C(Y(t_1) \overset{*}{-} P(t_1),\psi)}{\lambda}\right\}$$

from (13) we obtain (11). And this completes the proof of the theorem.

Corollary 1. *The ensemble of trajectories of system (9) is completely Y – controllable if* $\operatorname{rank} K = n$, *where* $K = (B, AB, ..., A^{n-1}B)$.

CONCLUSION

The problem of controllability is one of the important problems in the theory of optimal control. Here, for a controlled differential inclusion, the controllability problem was considered as the problem of the complete immersion of an ensemble of trajectories on a given terminal set. The goal of the study was set: clarification of the controllability conditions expressed in terms of the parameters of the system under consideration.

When studying the controllability ensemble trajectory control problem, the main conditions on the right-hand side of the considered differential inclusions are given in the form of assumptions 1-3. They cover a wide class of differential inclusion (1), as well as their linear models of the form (5) and (9).

Of the results obtained, the most general is Theorem 1. Theorem 2 gives necessary and sufficient conditions for "point" (x^0, Y) - controllability of the ensemble of trajectories linear in state of system (5).

Theorems 3 give the sufficient conditions of complete Y -controllability for the ensemble of trajectories for the linear model (9). These results generalize the known controllability conditions of linear systems [1] to the considered model of a dynamic control system. Note that Corollary 1 is an analogue of the well-known Kalman controllability criterion for system (9).

REFERENCES

1. Gabasov R., Kirillova F. M. Qualitative theory of optimal processes. Moskow, Nauka (1971).
2. Kurzhanskiy A.B. Control and observation under conditions of indeterminacy, Nauka, Moskow (1977).
3. Clarke F. Optimization and nonsmooth analysis, Willey & Sons, Ney York (1983).
4. Keyn V.N. Optimization of the control system by minimax criterion, Nauka, Moskow (1985).
5. Blagodatskikh V.I, Filippov A.F. Differential inclusions and optimal control, Trudy Mat. Inst, AN SSSR, vol.169 (1985). pp. 194-252.
6. Papageorgiou N.S. On the trajectories of controlled evolution inclusions, Comment. Math., Univ. Santi-Pauli. vol. 39, №1(1990). pp. 53-67.
7. Duda E.V., Minchenko L.I. On optimal trajectories of differential inclusions with delay. Differential equations. Vol. 33, No. 8 (1997).pp. 1023-1029.
8. Plotnikov A.B. The controlled kvazidifferential equations and its some property, Differens. Uravneniya, vol. № 10 (**1998). pp**. 1332-1336.
9. Otakulov S. About the conditions of controllability for differential inclusions, Izv. RAN, ser. Texn. Kibernetika, № 2 (1992). pp. 57–62.
10. Otakulov S., Sobirova G.D. About some properties of the set M-controllability for differential inclusions, Uzb. Math. Juorn., №1(2001). pp. 35–41.
11. Aseev S. M., An optimal control problem for a differential inclusion with a phase constraint. Smooth approximations and necessary optimality conditions, J. Math. Sci. (New York), vol. 103, № 6 (2001). pp. 670–685 .

12. Borosovich Yu.G, Gelman B.D, Mishkis A.D.,Obukhovskiy V.V. Introduction in theory multivalued maps and differential inclusions, КомKniga, Moskow. 2005.

13. Plotnikov A.V., Komleva T.A. Piecewise constant controlled linear fuzzy differential inclusions. Universal Journal of Applied Mathematics. vol.1, №2 (2013). pp. 39-43.

14. Israilov I, Kholiyarova F.Kh. Minimax control problem for differential inclusion with delayed argument. Materials of the international conference "Dynamic Systems: Stability, Control, Optimization" (DSSCO-2013). Minsk, October 1-5, 2013. pp. 139-141.

15. Poloninkin E.S. Set-valued analysis and differential inclusions, Fizmatlit, Moskow (2014).

16. Polovinkin E. S. Differential inclusions with unbounded right-hand side and necessary optimality conditions, Proc. Steklov Inst. Math., vol. 291 (2015), pp. 237-252.

17. Otakulov S. The control problem by ensemble of trajectories for differential inclusions, Lambert Academic Publishing, Riga (2019).

18. Otakulov S., Rahimov B. Sh. About conditions of controllability of ensamble trajectories of differential inclusion. Asian Journal of Multidimensional Research. Vol. 9, Issue 4(April 2020), Spl Issue 4. Pp. 205–213.